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RESULTS ON CARDINAL AND ORDINAL NUMBERS

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Abstract:- In this paper mainly we have obtained certain results on cardinal and ordinal numbers.

Introduction:- Cardinal numbers are basically those numbers which provide us an exact quantity of an object.In general on the set of real numbers, the relation '≤' is compatiable relation. In this paper mainly we obtained certain results on Cardinal and ordinal numbers.It is observed in result '1' that on the relation '≤' is also compatable on cardinal numbers. It is known that if a≤b then a+c≤b+d and a.c≤b.d it is also true on the set of cardinal numbers which is obtained in result 2; In general we have the elemantary result on the set of real numbers that $a^{(b+C)}=a^b.a^c$; $(ab)^c=a^c.b^c$, $a^{bc}=(ab)^c$. It is observed that the results holds for the set of cardinal numbers in Result 4. In result '5'We obtained that if a,b,c are cardinal numbers in which a≤b then $a^c≤b^c$; In general it is true for the set of real numbers that if 'a' is finite and 'b' is infinite then a+b is infinite which is also true for the set of cardinal numbers which is obtained in result 6. In general it is observed that if 'a' is infinite then a+a is infinite. If 'a' is an infinite cardinal number then a+a= a which is obtained in result 7. We also obtained necessary and sufficient condition on the set of ordinal numbers which is obtained in result 8.
First we start with the following definition;
Def 1: If a and b are cardinal numbers and if A and B are disjoint sets with card A = a and card B = b then the sum of the cardinal numbers a and b is a+b = card(AUB)
Def 2: If {ai} is a family of cardinal numbers and if {Ai} is the corresponding indexed family of pair wise disjoint sets with card Ai = ai for each i, then *E* ai = card(U Ai).
we have the following result.
Result 1: If a,b,c and D are cardinal numbers such that a < b and c <- d then a + b <= + d.
proof: we have card A = a , card B = b , card C = c and card D = d -> card A≤ card B ≥ A~ B.
since C ≤d =>card C ≤ card D=> c ~D
=>aUc ~ BUD
=> card(AUC)≤card (BUD)
=> a+c ≤ b+d
Def 3: if'a' and 'b' are any two cardinal numbers of the disjiont sets A and B with card A=a and card B=b then the product of the two cardinal numbers a and is
A.b=card(AXB)
If $\{a_i\}$ is the set of cardinal numbers of the disjiont sets $\{ A_i\}$ with card $A_i=\{a_i\}$. Then π_i a_i= card (xA_i)

RESULT 2: if a,b,c and d are cardinal numbers with a≤b and c≤d then a.c ≤b.d
PROOF: since a≤b=> card A ≤card B=>A~B
Since c≤d=> card C ≤card D=>C~D
=>AXC~BXD imply that a.c≤b.d.

RESULT3: if $\{a_i\}_{i\epsilon I}$ and $\{b_i\}_{i\epsilon I}$ are family of cardinal numbers such that $a_i<b_i\forall i\epsilon I$
Then E_i $a_i<\pi_i$ b_i
Proof:- let $\{A_i\}_{i\epsilon I}$ be indexed of family of the set with card $A_i=a_i$ and $\{B_i\}_{i\epsilon I}$ be indexed of family of

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the set with card $B_i=b_i$
Let $a_i<b_i$ =>cardA_i<card B_i
=>A_i~B_i =>U_iA_i~ U_iB_i
=>card(U_iA_i)<card(XB_i) imply that $E_ia_i<\pi_ib_i$
Def 4:-if a and b are any two cardinal numbers of the disjoint sets A and B with card A=a and card B =b then a^b=card (A^B)

Result 4 :- if a,b and c are cardinal numbers then
i.$a^{b+c}=a^b.a^c$
ii.$(ab)^c=a^c.b^c$
iii.$A^{bc}=(a^b)^c$
Proof:-Since,a,b,c,are the cardinal number there exisits disjiont sets A,B,C, with card A=a; card B=b and card C=c
Now, 1.a^{b+c} =card(A^{B+C})
=card(A^BA^C)
=card A^B.card A^C
=$a^b.a^c$
2. card(A^{B+c})=$\pi_{i\epsilon I}a_i$ where 'I' is the indexed set which has cardinality b+c.
$\Pi_{i\epsilon I'+i''=I}a_i=\pi_{i\epsilon I'}a_i.\ \pi_{i\epsilon I'}b_i$
where I' is the indexed set which has cardinality 'b' and I" is the indexed set which has cardinality 'C'
=$a^b.a^c$hence $a^{b+c}=a^b.a^c$
3.$(ab)^c$=card$(AB)^c=\pi_{i\epsilon I}a_i.b_i$ where 'I' is the indexed set which has cardinality 'c'=$\pi_{i\epsilon I}a_i.\ \pi_{i\epsilon I}b_i$ where I is the indexed set which has cardinality$a^{c.}\ b^c$
Hence $(ab)^c=a^c.b^c$
4. a^{bc}=card (A^{BC}) = $\pi_{i\epsilon I}a_i$
where I is the indexed set which has cardinality BC=$\pi_{i\epsilon I}a_i^{bi.ci}$
=the indexed set which has cardinality a^{bc}
=the indexed set which has cardinality $(a^b)^c$.
Now we have the following Definition
Def 5:- a cardniality number is said to be finite if it is the cardinal number of a finite set and is said to be infinite if it is the cardinal number of an infinite set.

Result 5:-If a,b,c are cardinal numbers such that a≤b then $a^c\leq b^c$
Proof:-
Let a,b,c be the cardinal number then there exists disjiont sets A,B and C with a=card A,b=card B,c=cardC
Since a≤b =>card A≤card B
=>A~B
=>A^C~B^C=>card A^C≤card$B^{C.}$

RESULT6:- IF a,and b are cardinal numbers such that 'a' is finite and 'b' is infinite then a+b=b.
Proof:-
Since a and b are cardinal numbers then there exists disjiont sets A and B such that card A=a and card B=b. Since 'a' in finite,the set 'A' is equivalent to some natural number
=>A~K
Since 'B' is infinite W~B
Now we have to define a map f:AUB->B
By,f/A:A->K

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f/W:n->n+k ∀n q.e.f(n)=n+k.∀n
f/B-W:x->x∀nϵB-W
hence there exists a one to one correspondence between AUB to B i.e.AUB~B
=>card(AUB)=card B Imply thata+b=b
RESULT7:-if ‘a’ is an infinite cardinal number then a+a=a.
Proof is trivial.
Now we have the

RESULT8:-if ɑ, β are distinct ordinal numbers then βϵɑ,iff β Cɑ and β is continuation of ɑ
Proof:-
Let αϵβ we claim that rϵα
Since rϵβ =>s(r)=r.
S(β)={ƌϵα=ƌ<β}
Since rϵβ=>rϵs(β)=>rϵα
Hence βCα.
Conversly suppose βCα, Since ‘α’ is an ordinal number ,s(α)=>α
=>βCs(α) => βϵα
Hence βϵα iff βCα
Define ≤by β≤α iff βCα or βϵα
Clearly ≤ in a patial order relation and for βϵα=>βCα as s(β)=β
We have s(β)={ƌϵα=ƌ<β}=β
=>’α’ is the initial segment continuation of β
Convervesely , ‘α’ is a continuation of β,
B={ƌϵα:ƌ<r}=s(r)
=> βCα=> βϵα
Hence β≤α iff ‘α’ is continuation of β.

RESULT 9:-Each well ordered set is similar of a unique ordinal number.
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