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4-TOTAL GEOMETRIC MEAN CORDIAL LABELING OF TREE RELATED GRAPHS

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Abstract :

Let G be a (p, q) graph. Let f : V (G) → {1, 2, 3,…,k} be a function where k∈N and k>1. For each edge uv, assign the label f (uv) = $\left\lceil \sqrt{f(u)f(v)} \right\rceil$. f is called k-Total geometric mean cordial labeling of G if | t_{mf} (i) – t_{mf} (j) | ≤ 1, for all i, j∈{1, 2, 3,…,k}, where t_{mf} (x) denotes the total number of vertices and edges labeled with x, x∈{1, 2, 3,…,k}. A graph that admits the k-total geometric mean cordial labeling is called k-total geometric mean cordial graph.

In this paper we investigate 4- total geometric mean cordial (4-TGMC) labeling behavior of some Tree related graphs.

Keywords:
Twig graph, Lily graph, Fire Crackers graph, W- graph, P(m, H_n) , 4- Total geometric mean cordial labeling, 4- TGMC graph.

I. Introduction

Graphs considered here are finite, simple and undirected. Graph labeling we refer to Gallian [1]. We pursue [2] for symbols and phrases. The motivation of the works done by [3], [4], [5], [6], [7] . The notation of k-total geometric mean cordial labeling of some graphs was introduced in [8]. We investigated 4-total geometric mean cordial labeling behavior of star graphs and disconnected graphs in [9], [10]. In this paper we investigate 4-total geometric mean cordial labeling of some Tree related graphs.

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Definition 1.1

Let G be a (p, q) graph. Let f : V (G) → {1, 2, 3,…,k} be a function where k∈N and k>1. For each edge uv, assign the label f (uv) = $\left\lceil \sqrt{f(u)f(v)} \right\rceil$. f is called k-Total geometric mean cordial labeling of G if | t_{mf} (i) – t_{mf} (j) | ≤ 1, for all i, j∈{1, 2, 3,…,k}, where t_{mf} (x) denotes the total number of vertices and edges labeled with x, x∈{1, 2, 3,…,k}. A graph that admits the k-total geometric mean cordial labeling is called k-total geometric mean cordial graph.

Definition 1.2.

The graph obtained from the path P_n by attaching exactly two pendant vertices to each internal vertex

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of path is called Twig graph.

Definition 1.3.

A lily graph l_n ,($n \geq 2$) can be constructed by two star graphs $2K_{1,n}$, $n \geq 2$ joining two path graphs $2P_n$, $n \geq 2$ with sharing a common vertex, that is $l_n = 2K_{1,n} + 2P_n$.

Definition 1.4.

Fire crackers graph is obtained by attaching a star graph S_m at each pendant vertices of P_n, such a fire cracker graph is denoted as $P_n \odot S_m$. Note that the fire cracker graph $P_n \odot S_m$ of order n+ 2m and of size n+ 2m -1.

Definition 1.5.

Consider the two copies of star graph $K_{1,n}$. If the last pendant vertex in the first copy of $K_{1,n}$ is merged with the initial pendant vertex in the second copy of $K_{1,n}$ then the resulting graph is called W-graph and it is denoted by W_{2n+1}. Clearly

$|E(W_{2n+1})| = 2n \text{ and } |V(W_{2n+1})| = 2n + 1.$

II. Main Results

Theorem 2.1.

Twig graph T_n is 4- total geometric mean cordial for all $n \geq 3$.

Proof:

Let $u_1, u_2, u_3, \dots, u_n$ be the vertices of the path P_n . Let v_i , w_i be the pendant vertices attached to the vertex u_i of P_n, which has 3n-4 vertices and 3n-5 edges.

The function f: V(G) → {1, 2, 3, 4} defined by

Case1: when n is odd

Allocate the label 3 to the $\frac{n+1}{2}$ vertices $u_1, u_2, u_3, \dots, u_{\frac{n+1}{2}}$, next we allocate the label 4 to the $\frac{n-1}{2}$ vertices $u_{\frac{n+3}{2}}, u_{\frac{n+5}{2}}, u_{\frac{n+7}{2}} \dots . u_n$ now we consider the pendant vertices, the labeling pattern of v and w is

Subcase 1: n ≡ 1 (mod 4)

Allocate the label 1 to the $\frac{3n-5}{2}$ vertices $v_1, v_2, v_3, \dots, v_{\frac{3n-3}{4}}$ *and* $w_1, w_2, w_3, \dots, w_{\frac{3n-7}{4}}$ and allocate the label 3 to the $\frac{n-3}{2}$ vertices $v_{\frac{3n+1}{4}}, v_{\frac{3n+5}{4}}, \dots, v_n$ *and* $w_{\frac{3n-3}{4}}, w_{\frac{3n+1}{4}}, w_{\frac{3n+5}{4}}, \dots, w_n$.

Subcase 2: n ≡ 3 (mod 4)

Now we allocate the label 1 to the $\frac{3n-5}{2}$ vertices $v_1, v_2, v_3, \dots, v_{\frac{3n-5}{4}}$ *and* $w_1, w_2, w_3, \dots, w_{\frac{3n-5}{4}}$, then we allocate the label 3 to the $\frac{n-3}{2}$ vertices $v_{\frac{3n-1}{4}}, v_{\frac{3n+3}{4}}, v_{\frac{3n+7}{4}}, \dots, v_n$ *and* $w_{\frac{3n-1}{4}}, w_{\frac{3n+3}{4}}, w_{\frac{3n+7}{4}}, \dots, w_n$.

Case 2: when n is even

Allocate the label 3 to the $\frac{n}{2}$ vertices $u_1, u_2, u_3, \dots, u_{\frac{n}{2}}$, next we allocate the label 4 to the $\frac{n}{2}$ vertices $u_{\frac{n+2}{2}}, u_{\frac{n+4}{2}}, u_{\frac{n+6}{2}}, \dots, u_n$, now consider the pendant vertices, the labeling pattern of v and w is

subcase 1: n ≡ 2 (mod 4)

Now we allocate the label 1 to the $\frac{3n-4}{2}$ vertices $v_1, v_2, v_3, \dots, v_{\frac{3n-2}{4}}$ *and* $w_1, w_2, w_3, \dots, w_{\frac{3n-6}{4}}$ then we allocate the label 3 to the $\frac{n-4}{2}$ vertices $v_{\frac{3n+2}{4}}, v_{\frac{3n+6}{4}}, \dots, v_n$ *and* $w_{\frac{3n-2}{4}}, w_{\frac{3n+2}{4}}, w_{\frac{3n+6}{4}}, \dots, w_n$.

subcase 2: n ≡ 4 (mod 4)

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Now we allocate the label 1 to the $\frac{3n-4}{2}$ vertices $v_1, v_2, v_3, \dots, v_{\frac{3n-4}{4}}$ *and* $w_1, w_2, w_3, \dots, w_{\frac{3n-4}{4}}$, then we allocate the label 3 to the $\frac{n-4}{2}$ vertices $v_{\frac{3n}{4}}, v_{\frac{3n+4}{4}}, v_{\frac{3n+8}{4}}, \dots, v_n$ *and* $w_{\frac{3n}{4}}, w_{\frac{3n+4}{4}}, w_{\frac{3n+8}{4}}, \dots, w_n$.
The vertex labeling f is 4- TGMC labeling

n	$t_{mf}(1)$	$t_{mf}(2)$	$t_{mf}(3)$	$t_{mf}(4)$
n *is odd*	n	n	n	$n-1$
n *is even*	$n-1$	n	n	n

From the above two cases we see that $| t_{mf}(i) - t_{mf}(j) | \leq 1$,for all $i, j \in \{1, 2, 3, \dots, 4\}$ hence twig graph is 4- total geometric mean cordial (4- TGMC) graph.

Theorem 2.2

For all $n \geq 2$, The lily graph l_n admits 4-TGMC labeling.

Proof:

Let $G = l_n$ be the lily graph with vertex set $V(G)= \{ x, x_i, y_i, s_j, t_j ; 1 \leq i \leq n-1, 1\leq j \leq n \}$. Let x be the centre vertex of G and x_i , y_i $(1 \leq i \leq n-1)$ be the vertices of first and second copy of path P_n, and s_j , t_j $(1 \leq j \leq n)$ be the vertices of first and second copy of star $K_{1,n}$, joining the n^{th} vertex of paths to the centre vertex of $K_{1, n}$.
Clearly $| V(G) | +| E(G) |= 8n-3$.
The function $f: V(G)\rightarrow \{1, 2, 3, 4\}$ defined by

$f(x) = 3$
$f(x_i) = 3 \quad 1 \leq i \leq n-1$
$f(y_i) = 4 \quad 1 \leq i \leq n-1$
$f(s_j) = 1 \quad 1 \leq j \leq n$
$f(t_j) = 1 \quad 1 \leq j \leq n-1$
$f(t_n) = 4.$

Clearly $t_{mf}(1)= t_{mf}(2)= t_{mf}(3)=2n-1$ and $t_{mf}(4)= 2n$.
The function f satisfies the condition $| t_{mf}(i) - t_{mf}(j) | \leq 1$,for all $i, j \in \{1, 2, 3, \dots, 4\}$ and hence l_n is 4-TGMC graph.

Theorem 2.3

For all $n \geq 2$, The fire crackers graph $P_n \Theta S_n$ admits 4-TGMC labeling.

Proof:

Let $G = P_n \Theta S_n$ be the fire crackers graph with vertex set $V(G)= \{ u_i, v_i, w_i ; 1 \leq i \leq n\}$and $E(G) = \{u_i u_{i+1}; 1 \leq i \leq n-1\} \cup \{ u_1 v_i, u_n w_i ; 1 \leq i \leq n\}$. Let u_i be the vertices of path P_n and v_i, w_i $(1 \leq i \leq n)$ be the pendant vertices attached to the end vertex of P_n. which has 3n vertices and 3n-1 edges.
The function $f: V(G)\rightarrow \{1, 2, 3, 4\}$ defined by
Case 1: when n is even

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$$f(u_i) = \begin{cases} 3, & 1 \le i \le \frac{n}{2} \\ 4, & \frac{n+2}{2} \le i \le n \end{cases}$$

$$f(v_i) = 1; \ 1 \le i \le n$$

$$f(w_i) = \begin{cases} 1, & 1 \le i \le \frac{n}{2} \\ 3, & \frac{n+2}{2} \le i \le n \end{cases}$$

Clearly $t_{mf}(1)= t_{mf}(2)= t_{mf}(4)= \frac{3n}{2}$ and $t_{mf}(3)= \frac{3n-2}{2}$.

Case 2: when n is odd

$$f(u_i) = \begin{cases} 3, & 1 \le i \le \frac{n+1}{2} \\ 4, & \frac{n+3}{2} \le i \le n \end{cases}$$

$$f(v_i) = 1; \ 1 \le i \le n$$

$$f(w_i) = \begin{cases} 1, & 1 \le i \le \frac{n-1}{2} \\ 3, & \frac{n+1}{2} \le i \le n \end{cases}.$$

Clearly $t_{mf}(1)= t_{mf}(2)= t_{mf}(4)= \frac{3n-1}{2}$ and $t_{mf}(3)= \frac{3n+1}{2}$.

In the above cases we see that $| t_{mf}(i) - t_{mf}(j) | \le 1$, for all $i, j \in \{1, 2, 3, \ldots, 4\}$ and hence $P_n \odot S_n$ is 4-TGMC graph.

Theorem 2.4.

The W graph W_{2n+1} is 4-total geometric mean cordial graph for $n \ge 2$.

Proof

Consider the first and second copies of the star graph $K^1_{1,n}$ and $K^2_{1,n}$ with an apex vertex x and y. Let $V(K^1_{1,n}) = \{ x, s_1, s_2, s_3, \ldots s_n \}$ and $V(K^2_{1,n}) = \{ y, t_1, t_2, t_3, \ldots t_n \}$.The graph obtained by adjoining s_n and t_1. Let G be the W graph with 2n+1 vertices and 2n edges.

The function $f : V(G) \to \{1, 2, 3, 4 \}$ defined by

$f(x) = 1$, $f(y) = 4$

$$f(s_i) = \begin{cases} 1, & 1 \le i \le \frac{n-1}{2} \text{ if n is odd} \\ & 1 \le i \le \frac{n}{2} \text{ if n is even} \\ 2 & \frac{n+1}{2} \le i \le n \text{ if n is odd} \\ & \frac{n+2}{2} \le i \le n \text{ if n is even} \end{cases}$$

$f(t_i) = 3$, $2 \le i \le n$.

Then we get $|t_{mf}(i) - t_{mf}(j)| \le 1$ for all $i, j \in \{1, 2, 3, \ldots, 4\}$.

Hence the function f is 4-total geometric mean cordial labeling and G is 4-TGMC graph.

Theorem 2.5.

The graph $P(m, H_n)$ is 4- total geometric mean cordial labeling for all $m \ge 2$, $n \ge 3$.

Proof:

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Let G = P(m, H_n), m ≥ 2, n ≥ 3 be a graph with vertex set and edge sets are
V(P(m, H_n)) = $\{ u_i{}^j\ v_i{}^j, 1 \le i \le n; 1 \le j \le m\}$ and
E(P(m, H_n)) = $\{ u_i{}^j u_{i+1}{}^j, v_i{}^j v_{i+1}{}^j, 1 \le i \le n-1; 1 \le j \le m\} \cup$
$\{u^j{}_{\frac{n+1}{2}}\ v^j{}_{\frac{n+1}{2}}, if\ n\ is\ odd;\ \ u^j{}_{\frac{n+2}{2}}\ v^j{}_{\frac{n}{2}}, if\ n\ is\ even\} \cup \{v_1{}^j\ v_1{}^{j+1}, 1 \le j \le m-1\}$
Therefore the graph G has $\left|V\left(P(m,H_n)\right)\right| + \left|E\left(P(m,H_n)\right)\right| = 4mn - 1$
The function f: V(G)= { 1, 2, 3, 4} defined by the following two cases
Case 1: when n is odd

$$f(u_i{}^j) = \{ 1 \quad 1 \le i \le \frac{n+1}{2};\quad 1 \le j \le m$$
$$2 \quad \frac{n+3}{2} \le i \le n;\quad 1 \le j \le m\}$$
$$f(v_i{}^j) = \{ 4 \quad 1 \le i \le \frac{n-1}{2};\quad 1 \le j \le m$$
$$3 \quad \frac{n+1}{2} \le i \le n;\quad 1 \le j \le m\}$$

Case 2: when n is even

$$f(u_i{}^j) = \{ 1 \quad 1 \le i \le \frac{n}{2};\quad 1 \le j \le m$$
$$2 \quad i = \frac{n+2}{2};\quad 1 \le j \le m$$
$$3 \quad \frac{n+4}{2} \le i \le n;\quad 1 \le j \le m\}$$
$$f(v_i{}^j) = \{ 4 \quad 1 \le i \le \frac{n}{2};\quad 1 \le j \le m$$
$$2 \quad \frac{n+2}{2} \le i \le n-1;\quad 1 \le j \le m$$
$$1 \quad i = n\}$$

Clearly $t_{mf}(1) = t_{mf}(2) = t_{mf}(3)$ = mn and $t_{mf}(4)$ = mn-1
The above two cases the function f satisfies the condition| $t_{mf}(i) - t_{mf}(j)$ | ≤ 1, for all i, j∈{1, 2, 3,…,4}, and hence P(m, H_n) is 4-TGMC graph.

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