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NEW EXPONENTIAL LINEAR FAILURE RATE DISTRIBUTION: PROPERTIES AND APPLICATION TO GLASS FIBERS DATA

Alshad K B[1*], Arul Chezhian A[2]

[1*,2]Sri Ramachandra Faculty of Engineering and Technology, Sri Ramachandra Institute of Higher Education and Research (Category -1 Deemed to be University), Porur, Chennai–600116, Tamilnadu, India.

Abstract. This manuscript introduces a generalization of the linear failure rate (LFR) distribution, called the new exponential linear failure rate (NExLFR) distribution, with an application to the strengths of glass fibers. The new model is highly flexible and can be effectively applied in various fields. We present several mathematical properties of the new model, including the density function, cumulative distribution function, survival function, and hazard rate function of the NExLFR distribution. Method of maximum likelihood estimation is discussed to estimate the unknown parameters of the model. Finally, to showcase the flexibility of this distribution and for illustrative purposes, we apply it to a real dataset and NExLFR shows perfect fit for the strengths of glass fibers data.

1.INTRODUCTION

During the past decade, various statistical distributions have been applied in several fields such as insurance, epidemiology, and engineering. However, in many practical situations, standard statistical models are not suitable for modeling real-world problems. This has led to the development of generalized distributions. In this article, we discuss a new distribution called New Exponential Linear Failure Rate (NExLFR) distribution, characterized by three parameters: μ, α and θ.

The Linear Failure Rate (LFR) distribution, also known as the linear exponential distribution, is a versatile model useful for analyzing lifetime data. The probability density function (PDF) and cumulative distribution function (CDF) of the LFR distribution with parameters $\boldsymbol{\alpha}, \boldsymbol{\theta} > 0$ are given by:

$$f(x;\theta,\alpha) = (\alpha + \theta x)e^{-\left(\alpha x + \frac{\theta}{2}x^2\right)} \quad \text{----------(1)}$$

$$F(x;\theta,\alpha) = 1 - e^{-\left(\alpha x + \frac{\theta}{2}x^2\right)} \quad \text{----------(2)}$$

Generalizing probability distributions is a common and appropriate practice in statistical research. Many researchers have developed generalized and extended versions of the LFR model, such as the generalized LFR exponential distribution, truncated Cauchy power LFR distribution, transmuted LFR distribution, exponentiated GLFR distribution, modified beta generalized LFR distribution etc. This paper introduces a novel distribution, the new exponential linear failure rate (NExLFR) model, by incorporating the Linear Failure Rate (LFR) distribution into the new exponential family of distribution introduced by Farrukh et al. The motivation behind adopting the new exponential family of distributions lies in its capacity to enhance the flexibility of the LFR distribution. The CDF and PDF are given respectively as:

$$G(t) = 1 - exp\left\{-a\left[\frac{F(t;\omega)\left(2-F(t;\omega)\right)}{1-F(t;\omega)}\right]\right\}, a > 0 \quad \text{----------(3)}$$

$$g(t) = \frac{a\, f(t;\omega)}{[\,1-F(t;\omega)]}\{\,1 + [1 - F(t;\omega)]^2\}\, exp\left\{-a\left[\frac{F(t;\omega)\left(2-F(t;\omega)\right)}{1-F(t;\omega)}\right]\right\} \quad \text{----------(4)}$$

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where $F(t;\omega)$ and $f(t;\omega)$ denotes the baseline *cdf* and *pdf* respectively.
The rest of the manuscript is prepared as follows. Section 2 introduces the PDF and CDF of the NExLFR model. In Section 3, we derive the survival function and hazard rate function of the proposed model. Maximum likelihood estimation described in Section 4, and the applicability of the model using a real dataset is presented in Section 5 and finally, section 6 concludes the paper.

2. NEW EXPONENTIAL LINEAR FAILURE RATE

A random variable X is said to have the new exponential linear failure rate distribution with parameter μ, α and θ if its PDF is defined as:

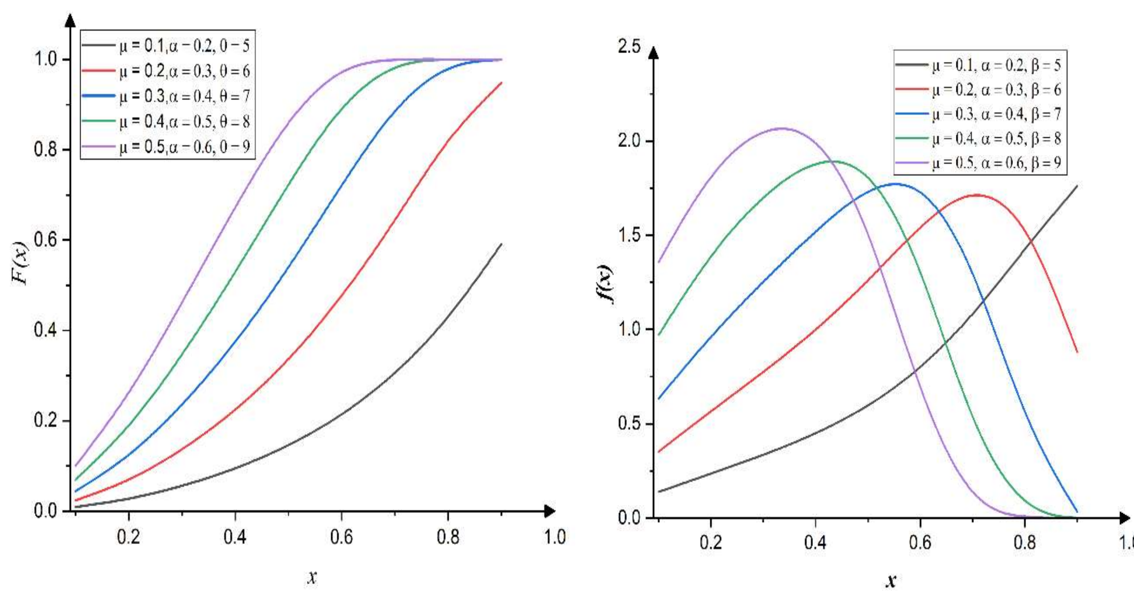
$$\boldsymbol{f(x) = \mu(\alpha + \theta x)e^{(\varphi)}\left(1 + e^{(-2(\varphi))}\right)e^{\left(-\mu\left(e^{(\varphi)} - e^{(-\varphi)}\right)\right)}} \quad \text{----------(5)}$$

and its CDF is

$$\boldsymbol{F(x) = 1 - e^{\left(-\mu\left(e^{(\varphi)} - e^{(-\varphi)}\right)\right)}} \quad \text{----------(6)}$$

α and θ are two scale and μ is the location parameters and $\varphi(x;\mu,\alpha,\theta) = \varphi = \alpha x + \frac{\theta}{2}x^2$

Figure 1: Plot for PDF and CDF of NExLFR distribution for different values of parameters.



3. SURVIVAL FUNCTION AND HAZARD RATE FUNCTION

The survival function ($\boldsymbol{S(x)}$) and hazard rate function ($\boldsymbol{H(x)}$) of NExLFR is given by respectively.

$$\boldsymbol{S(x) = e^{\left(-\alpha\left(e^{(\varphi)} - e^{(-\varphi)}\right)\right)}} \quad \text{----------(7)}$$

$$\boldsymbol{H(x) = \frac{\alpha(\alpha+\theta x)e^{(\varphi)}(1+e^{(-2\varphi)})e^{\left(-\alpha\left(e^{(\varphi)}-e^{(-\varphi)}\right)\right)}}{e^{\left(-\alpha\left(e^{(\varphi)}-e^{(-\varphi)}\right)\right)}}} \quad \text{----------(8)}$$

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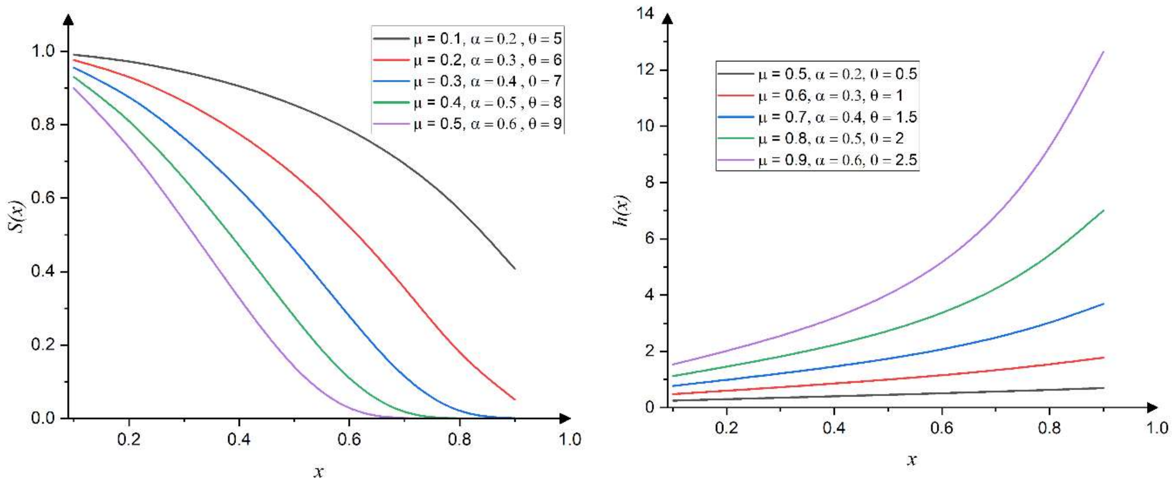
Figure 2: Plot for the hazard rate function and survival function of NExLFR distribution for different values of parameters.



4. MAXIMUM LIKELIHOOD ESTIMATION

In this section, we propose the problem of estimating the unknown parameters μ, θ and α of the NExLFR distribution using maximum likelihood estimation. Let $x_1, x_2, \ldots, x_n$ is a random sample of size *n* taken from the NExLFR distribution. The log-likelihood function is given by

$$log\,L = n\,log\,a + \sum_{i=1}^{n}\left(\alpha x_i + \frac{\theta x_i^2}{2}\right) + \left\{-a\sum_{i=1}^{n}\left[e^{\left(\alpha x_i+\frac{\theta x_i^2}{2}\right)} - e^{-\left(\alpha x_i+\frac{\theta x_i^2}{2}\right)}\right]\right\} + \sum_{i=1}^{n} log\left[1 + e^{-2\left(\alpha x_i+\frac{\theta x_i^2}{2}\right)}\right](\alpha + \theta x_i) \quad \text{----------(9)}$$

Differentiating equation (9) with respect to μ, θ and α and equating to zero gives

$$\frac{\partial\, log\, L}{\partial \mu} = \frac{n}{a} - \sum_{i=1}^{n}\left[e^{\left(\alpha x_i+\frac{\theta x_i^2}{2}\right)} - e^{-\left(\alpha x_i+\frac{\theta x_i^2}{2}\right)}\right] \quad \text{----------(10)}$$

$$\frac{\partial\, log\, L}{\partial \alpha} = \sum_{i=1}^{n} x_i - a\sum_{i=1}^{n}\left(e^{-\left(\alpha x_i+\frac{\theta x_i^2}{2}\right)}x_i + e^{\alpha x_i+\frac{\theta x_i^2}{2}}x_i\right) + \sum_{i=1}^{n}\left\{Log\left(1 + e^{-2\alpha x_i+\frac{\theta x_i^2}{2}}\right) - \frac{2e^{-2\alpha x_i+\frac{\theta x_i^2}{2}}x_i(\alpha+\theta x_i)}{1+e^{-2\alpha x_i+\frac{\theta x_i^2}{2}}}\right\} \quad ---(11)$$

$$\frac{\partial\, log\, L}{\partial \theta} = \sum_{i=1}^{n}\frac{x_i^2}{2} - a\sum_{i=1}^{n}\left(\frac{1}{2}e^{-\left(\alpha x_i+\frac{\theta x_i^2}{2}\right)}x_i^2 + \frac{1}{2}e^{\alpha x_i+\frac{\theta x_i^2}{2}}x_i^2\right) + \sum_{i=1}^{n}\left\{Log\left(1 + e^{-2\alpha x_i+\frac{\theta x_i^2}{2}}\right) + \right.$$

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$$\left.\frac{e^{-2\alpha x_i+\frac{\theta x_i^2}{2}}(\alpha+\theta x_i)}{2\left(1+e^{-2\alpha x_i+\frac{\theta x_i^2}{2}}\right)}\right\} - - - (12)$$

Since the equations (10), (11) and (12) cannot be solved analytically, some numerical methods like the Newton– Raphson algorithm can be used to obtain the estimates of the model parameters, μ, θ *and* α.

5. APPLICATION

The data set consists of 63 observations of the strengths of 1.5 cm glass fibres collected at the National Physical Laboratory in England and published by Smith and Naylor. A number of goodness-of-fit metrics, including log-likelihood (-2l), Akaike information criterion (AIC), corrected Akaike information criterion (CAIC), Bayesian information criterion (BIC), and Hannan-Quinn criterion (HQC), are used to compare the competitive models, including NExW (Jamal, Farrukh, et al.), APW (Nassar et al.), EW (Nadarajah et al.), IW (De Gusmao et al.), OBIIIW (Usman et al.), and TLIW (Abbas et al.). From Table 2, it is clear that the NExLFR distribution consistently produces the lowest values for the criteria mentioned above. Consequently, we can conclude that the proposed distribution offers the greatest fit for the given data sets

Table 1: The MLEs and corresponding SEs (in parentheses) for data.

Distribution	α	β	μ	γ	c	θ	k
NExLFR	1.2142 (0.2263)	-	0.4332 (0.1488)	-	-	2.2411 (0.4187)	-
NExW	0.2011 (0.3187)	2.1775 (2.835)	0.2743 (0.1005)	-	-		-
APW	15.9295 (9.0403)	-2.1408 (0.2014)	3.4831 (0.3878)	-	-		-
EW	0.7219 (0.3612)	7.1702 (2.2189)	0.5858 (0.0388)	-	-		-
IW	-	1.9695 (0.2486)	-	2.8857 (0.2343)	-		-
OBIIIW	0.0145 (0.0223)	5.3505 (0.7807)	-	-	6.4685 (4.0106)		2.5220 (1.1742)
TLIW	0.6819 (0.4159)	3.8264 (1.8774)	-	2.4243 (0.2574)	-		-

Table 2: −2l, AIC, AICC, BIC for the data set .

Distribution	*-2l*	**AIC**	**AICC**	**BIC**	**HQC**
NExLFR	**5.75890**	**17.5178**	**17.822**	**23.94721**	**20.0465**
NExW	11.5352	31.0705	31.7601	39.6430	34.4421
APW	43.2036	92.4073	92.8141	98.8367	94.9360
EW	14.7319	35.4638	35.8705	41.8932	37.9925
IW	46.8533	97.7067	97.9067	101.9930	99.3925
OBIIIW	15.6037	37.2074	37.6141	43.6367	39.7361
TLIW	39.5849	85.1699	85.5767	91.5993	87.6986

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6. Conclusion

In this paper, we introduced a new probability distribution and named it new exponential linear Failure Rate Distribution. We derived its characteristics, including the hazard rate function and survival function. Its parameters have been estimated through a discussion of maximum likelihood estimation. An example of glass fibers strengths has been used to illustrate the applications of the distribution and concluded that the NExLFR distribution offers the best fit for the given data sets.

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