CAHIERS MAGELLANES-NS
Volume 06 Issue 2
2024

ISSN:1624-1940
DOI 10.6084/m9.figshare.26310216
http://magellanes.com/

TOPOLOGICAL OPERATORS OVER BIPOLAR INTUITIONISTIC FUZZY α-IDEAL OF A BP-ALGEBRA

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Abstract:
The idea of an interior operator and a new algebraic structure of BP-algebra is the concept of a bipolar intuitionistic fuzzy α-ideal. This study aims to apply the ideal theory and fuzzy set theory of a BP-algebra. The relationships between the topological operators' operations on the bipolar intuitionistic fuzzy α-ideal are determined.

Keywords: BP-algebra, bipolar fuzzy ideal, bipolar intuitionistic fuzzy α-ideal, interior operator.

1. Introduction

Since L.A. Zadeh [8] first proposed the concept of fuzzy sets, research in graph theory, engineering, and medical science has continued. A.S. Ahn is credited with creating BP-algebra. In 2012, BP-Algebras were initially presented by Sun Shin Ahn and Jeong Soon Han [2]. Based on K.J. Lee's description [4], bipolar valued fuzzy sets are an extension of fuzzy sets with a wider positive membership degree range, from [0, 1] to [-1, 1]. A bipolar valued fuzzy set is associated with three membership degrees: 0 indicates that the elements are not significant to the relevant property, 1 indicates that the items partially satisfy the property, and -1 indicates that the components partially satisfy the implicit counter property. In bipolar valued fuzzy sets, elements with a membership degree of 0 indicate that the corresponding property is not affected; elements with a membership degree of positive (0, 1) indicate partial property satisfaction; and elements with a membership degree of negative (-1, 0) indicate partial satisfaction of the implicit counter property. A generalization of fuzzy sets are bipolar fuzzy sets, which were first described in 1994 by scholar W.R. Zhang [9]. The union and intersection of intuitionistic fuzzy sets was the subject of an analysis by K. Chakrabarty and Biswas R. Nanda [3]. Level subgroups and fuzzy groups were examined for A. Rajesh Kumar [7]. The concept of fuzzy sets was developed into intuitionistic fuzzy sets by Atanassov in 1986. The notion and numerous operations of intuitionistic fuzzy primary and semiprimary ideals were established by M. Palanivelrajan and S. Nandakumar [6]. Many facets are examined in the development of the concepts of bipolar fuzzy sub LA-semigroup and bipolar fuzzy left (right) ideal of LA-semigroup by S. Abdullah [1] and M.M.M. Aslam. A few characterization theorems for bipolar fuzzy left (right) ideals of LA-semigroups are also provided. There are several types of LA-semigroups characterized by bipolar fuzzy ideals. The bipolar

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fuzzy α-ideal of BP algebra was defined in 2020 by Osama Rashad EI-Gendy [5].

2020 Mathematics Subject Classification. 03E72. Key words and phrases. Theory of Fuzzy sets, etc.

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2. Preliminaries

Definition: 2.1

Given any two bipolar intuitionistic fuzzy sets, let A = $(\chi_{A}^{+}, \chi_{A}^{-}, \psi_{A}^{+}, \psi_{A}^{-})$ and B = $(\chi_{B}^{+}, \chi_{B}^{-}, \psi_{B}^{+}, \psi_{B}^{-})$ in Γ, we define

a. A ∩ B={$(\kappa$, min$(\chi_{A}^{+}(\kappa), \chi_{B}^{+}(\kappa))$, max$(\chi_{A}^{-}(\kappa), \chi_{B}^{-}(\kappa))$, max$(\psi_{A}^{+}(\kappa), \psi_{B}^{+}(\kappa))$, min$(\psi_{A}^{-}(\kappa), \psi_{B}^{-}(\kappa)))$ / $\kappa \in \Gamma$ }

b. A ∪ B ={(κ , max$(\chi_{A}^{+}(\kappa), \chi_{B}^{+}(\kappa))$, min$(\chi_{A}^{-}(\kappa), \chi_{B}^{-}(\kappa))$, min$(\psi_{A}^{+}(\kappa), \psi_{B}^{+}(\kappa))$, max$(\psi_{A}^{-}(\kappa), \psi_{B}^{-}(\kappa)))$ / $\kappa \in \Gamma$ }

c. $\overline{A}$ = {(κ ,$\psi_{A}^{+}(\kappa), \psi_{A}^{-}(\kappa), \chi_{A}^{+}(\kappa), \chi_{A}^{-}(\kappa))$ / $\kappa \in \Gamma$ }.

Definition: 2.2

BP-algebra's bipolar intuitionistic fuzzy set A = $\{\chi_{A}^{+}, \chi_{A}^{-}, \psi_{A}^{+}, \psi_{A}^{-}$ / $\kappa \in \Gamma$ }, If Γ meets the following criteria, it is referred to as a bipolar intuitionistic fuzzy α-ideal of Γ:

a. $\chi_{A}^{+}(0) \geq \chi_{A}^{+}(\kappa)$ and $\chi_{A}^{-}(0) \leq \chi_{A}^{-}(\kappa)$

b. $\chi_{A}^{+}(\lambda * \mu) \geq \min\{ \chi_{A}^{+}(\kappa * \mu),\ \chi_{A}^{+}(\kappa * \lambda)\}$

c. $\chi_{A}^{-}(\lambda * \mu) \leq \max\{ \chi_{A}^{-}(\kappa * \mu),\ \chi_{A}^{-}(\kappa * \lambda)\}$

d. $\psi_{A}^{+}(0) \leq \psi_{A}^{+}(\kappa)$ and $\psi_{A}^{-}(0) \geq \psi_{A}^{-}(\kappa)$

e. $\psi_{A}^{+}(\lambda * \mu) \leq \max\{ \psi_{A}^{+}(\kappa * \mu),\ \psi_{A}^{+}(\kappa * \lambda) \}$

f. $\psi_{A}^{-}(\lambda * \mu) \geq \min\{ \psi_{A}^{-}(\kappa * \mu),\ \psi_{A}^{-}(\kappa * \lambda) \}$, for all $\kappa, \lambda, \mu \in \Gamma$.

Definition: 2.3

Let A is a bipolar intuitionistic fuzzy set of Γ, then the interior operator $\mathcal{I}$ is defined by $\mathcal{I}$ (A) = {(κ, min $\chi_{A}^{+}(\lambda)$, max$\chi_{A}^{-}(\lambda)$, max $\psi_{A}^{+}(\lambda)$, min $\psi_{A}^{-}(\lambda))$ / $\kappa \in \Gamma$, $\lambda \in \Gamma$ }.

Definition: 2.4

Let A is a bipolar intuitionistic fuzzy set of Γ, then the necessity operator □ is defined by □ A = {($\kappa, \chi_{A}^{+}(\kappa), \chi_{A}^{-}(\kappa), 1 - \chi_{A}^{+}(\kappa), -1 - \chi_{A}^{-}(\kappa))$ / $\kappa \in \Gamma$}.

Definition: 2.5

Let A is a bipolar intuitionistic fuzzy set of Γ, then the possibility operator ◊ is defined by ◊ A = { $(\kappa, 1 - \psi_{A}^{+}(\kappa), -1 - \psi_{A}^{-}(\kappa), \psi_{A}^{+}(\kappa), \psi_{A}^{-}(\kappa)$) / $\kappa \in \Gamma$}.

3. Results and Discussion

Theorem: 3.1

If A is a bipolar intuitionistic fuzzy α-ideal of Γ, then $\mathcal{I}$(A) is a bipolar intuitionistic fuzzy α-ideal of Γ.

Proof: Given A is a bipolar intuitionistic fuzzy α-ideal of Γ.

Consider 0, κ, λ, μ ∈ A.

a) Now $\chi_{\mathcal{I}(A)}^{+}(0)$ = min $\chi_{A}^{+}(\kappa) = \chi_{A}^{+}(a) \geq$ min $\chi_{A}^{+}(a) = \chi_{\mathcal{I}(A)}^{+}(\kappa)$.

Therefore $\chi_{\mathcal{I}(A)}^{+}(0) \geq \chi_{\mathcal{I}(A)}^{+}(\kappa)$.

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http://magellanes.com/

Now $\chi^-_{\mathcal{I}(A)}(0) = \max\ \chi^-_A(\kappa) = \chi^-_A(a) \le \max\ \chi^-_A(a) = \chi^-_{\mathcal{I}(A)}(\kappa)$.
Therefore $\chi^-_{\mathcal{I}(A)}(0) \le \chi^-_{\mathcal{I}(A)}(\kappa)$.
b) Now $\chi^+_{\mathcal{I}(A)}(\lambda * \mu) = \min\ \chi^+_A(b * c) \ge \min\ \{ \min\ \{ \chi^+_A(a * c),\ \chi^+_A(a * b)\}\}$
$= \min\ \{ \min \chi^+_A(a * c),\ \min \chi^+_A(a * b)\}$
$= \min\ \{ \chi^+_{\mathcal{I}(A)}(\kappa * \mu),\ \chi^+_{\mathcal{I}(A)}(\kappa * \lambda)\}$.
Therefore $\chi^+_{\mathcal{I}(A)}(\lambda * \mu) \ge \min\ \{ \chi^+_{\mathcal{I}(A)}(\kappa * \mu),\ \chi^+_{\mathcal{I}(A)}(\kappa * \lambda)\}$.
c) Now $\chi^-_{\mathcal{I}(A)}(\lambda * \mu) = \max\ \chi^-_A(b * c) \le \max\ \{ \max\ \{ \chi^-_A(a * c),\ \chi^-_A(a * b)\}\}$
$= \max\ \{ \max\ \chi^-_A(a * c),\ \max \chi^-_A(a * b)\}$
$= \max\ \{ \chi^-_{\mathcal{I}(A)}(\kappa * \mu),\ \chi^-_{\mathcal{I}(A)}(\kappa * \lambda)\}$.

Therefore $\chi^-_{\mathcal{I}(A)}(\lambda * \mu) \le \max\ \{ \chi^-_{\mathcal{I}(A)}(\kappa * \mu),\ \chi^-_{\mathcal{I}(A)}(\kappa * \lambda)\}$.
d) Now $\psi^+_{\mathcal{I}(A)}(0) = \max \psi^+_A(\kappa) = \psi^+_A(a) \le \max \psi^+_A(a) = \psi^+_{\mathcal{I}(A)}(\kappa)$.
Therefore $\psi^+_{\mathcal{I}(A)}(0) \le \psi^+_{\mathcal{I}(A)}(\kappa)$.
Now $\psi^-_{\mathcal{I}(A)}(0) = \min \psi^-_A(\kappa) = \psi^-_A(a) \ge \min \psi^-_A(a) = \psi^-_{\mathcal{I}(A)}(\kappa)$.
Therefore $\psi^-_{\mathcal{I}(A)}(0) \ge \psi^-_{\mathcal{I}(A)}(\kappa)$.
e) Now $\psi^+_{\mathcal{I}(A)}(\lambda * \mu) = \max \psi^+_A(b * c) \le \max\ \{ \max\ \{ \psi^+_A(a * c),\ \psi^+_A(a * b)\}\}$
$= \max\ \{ \max \psi^+_A(a * c),\ \max \psi^+_A(a * b)\}$
$= \max\ \{ \psi^+_{\mathcal{I}(A)}(\kappa * \mu),\ \psi^+_{\mathcal{I}(A)}(\kappa * \lambda)\}$.

Therefore $\psi^+_{\mathcal{I}(A)}(\lambda * \mu) \le \max\ \{ \psi^+_{\mathcal{I}(A)}(\kappa * \mu),\ \psi^+_{\mathcal{I}(A)}(\kappa * \lambda)\}$.
f) Now $\psi^-_{\mathcal{I}(A)}(\lambda * \mu) = \min \psi^-_A(b * c) \ge \min\ \{ \min\ \{ \psi^-_A(a * c),\ \psi^-_A(a * b)\ \}\}$
$= \min\ \{ \min \psi^-_A(a * c),\ \min \psi^-_A(a * b)\}$
$= \min\ \{ \psi^-_{\mathcal{I}(A)}(\kappa * \mu),\ \psi^-_{\mathcal{I}(A)}(\kappa * \lambda)\}$.
Therefore $\psi^-_{\mathcal{I}(A)}(\lambda * \mu) \ge \min\ \{ \psi^-_{\mathcal{I}(A)}(\kappa * \mu),\ \psi^-_{\mathcal{I}(A)}(\kappa * \lambda)\}$.
Therefore $\mathcal{I}(A)$ is a bipolar intuitionistic fuzzy α-ideal of Γ.

Theorem: 3.2
If A is a bipolar intuitionistic fuzzy α-ideal of Γ, then $\mathcal{I}(\mathcal{I}(A)) = \mathcal{I}(A)$ is a bipolar intuitionistic fuzzy α-ideal of Γ.

Proof: Given A is a bipolar intuitionistic fuzzy α-ideal of Γ.
Consider $0, \kappa, \lambda, \mu \in A$.
a) Now $\chi^+_{\mathcal{I}(\mathcal{I}(A))}(0) = \min\ \chi^+_{\mathcal{I}(A)}(\kappa) = \min\ (\min \chi^+_A(0)) \ge \min\ \chi^+_A(0) = \chi^+_{\mathcal{I}(A)}(\kappa)$.
Therefore $\chi^+_{\mathcal{I}(\mathcal{I}(A))}(0) \ge \chi^+_{\mathcal{I}(A)}(\kappa)$.
Now $\chi^-_{\mathcal{I}(\mathcal{I}(A))}(0) = \max\ \chi^-_{\mathcal{I}(A)}(\kappa) = \max\ (\max \chi^-_A(0)) \le \max\ \chi^-_A(0) = \chi^-_{\mathcal{I}(A)}(\kappa)$.
Therefore $\chi^-_{\mathcal{I}(\mathcal{I}(A))}(0) \le \chi^-_{\mathcal{I}(A)}(\kappa)$.

b) Now $\chi^+_{\mathcal{I}(\mathcal{I}(A))}(\lambda * \mu) = \min\ \chi^+_{\mathcal{I}(A)}(b * c) = \min\ (\min \chi^+_A(\lambda * \mu)) = \min\ (\chi^+_A(b * c))$
$\ge \min\ \{ \min\ \{ \chi^+_A(a * c),\ \chi^+_A(a * b)\}\}$
$= \min\ \{ \min \chi^+_A(a * c),\ \min \chi^+_A(a * b)\}$
$= \min\ \{ \chi^+_{\mathcal{I}(A)}(\kappa * \mu),\ \chi^+_{\mathcal{I}(A)}(\kappa * \lambda)\}$.
Therefore $\chi^+_{\mathcal{I}(\mathcal{I}(A))}(\lambda * \mu) \ge \min\ \{ \chi^+_{\mathcal{I}(A)}(\kappa * \mu),\ \chi^+_{\mathcal{I}(A)}(\kappa * \lambda)\}$.

CAHIERS MAGELLANES-NS
Volume 06 Issue 2
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ISSN:1624-1940
DOI 10.6084/m9.figshare.26310216
http://magellanes.com/

c) Now $\chi^{-}_{\mathcal{I}(\mathcal{I}(A))}(\lambda * \mu)$ = max $\chi^{-}_{\mathcal{I}(A)}(b * c)$ = max(max $\chi^{-}_{A}(\lambda * \mu)$)= max ($\chi^{-}_{A}(b * c)$)
$\leq$ max { max { $\chi^{-}_{A}(a * c),\ \chi^{-}_{A}(a * b)$}}
= max { max $\chi^{-}_{A}(a * c)$, max $\chi^{-}_{A}(a * b)$}
= max { $\chi^{-}_{\mathcal{I}(A)}(\kappa * \mu),\ \chi^{-}_{\mathcal{I}(A)}(\kappa * \lambda)$}.

Therefore $\chi^{-}_{\mathcal{I}(\mathcal{I}(A))}(\lambda * \mu) \leq$ max { $\chi^{-}_{\mathcal{I}(A)}(\kappa * \mu),\ \chi^{-}_{\mathcal{I}(A)}(\kappa * \lambda)$}.

d) Now $\psi^{+}_{\mathcal{I}(\mathcal{I}(A))}(0)$= max $\psi^{+}_{\mathcal{I}(A)}(\kappa)$ = max (max $\psi^{+}_{A}(0)$) $\leq$ max $\psi^{+}_{A}(0) = \psi^{+}_{\mathcal{I}(A)}(\kappa)$.
Therefore $\psi^{+}_{\mathcal{I}(\mathcal{I}(A))}(0) \leq \psi^{+}_{\mathcal{I}(A)}(\kappa)$.
Now $\psi^{-}_{\mathcal{I}(\mathcal{I}(A))}(0)$ = min $\psi^{-}_{\mathcal{I}(A)}(\kappa)$ = min (min $\psi^{-}_{A}(0)$) $\geq$ min $\psi^{-}_{A}(0) = \psi^{-}_{\mathcal{I}(A)}(\kappa)$.
Therefore $\psi^{-}_{\mathcal{I}(\mathcal{I}(A))}(0) \geq \psi^{-}_{\mathcal{I}(A)}(\kappa)$.

e) Now $\psi^{+}_{\mathcal{I}(\mathcal{I}(A))}(\lambda * \mu)$= max $\psi^{+}_{\mathcal{I}(A)}(b * c)$ = max (max $\psi^{+}_{A}(\lambda * \mu)$) = max ($\psi^{+}_{A}(b * c)$)
$\leq$ max { max { $\psi^{+}_{A}(a * c),\ \psi^{+}_{A}(a * b)$}}
= max { max $\psi^{+}_{A}(a * c)$, max $\psi^{+}_{A}(a * b)$}
= max { $\psi^{+}_{\mathcal{I}(A)}(\kappa * \mu),\ \psi^{+}_{\mathcal{I}(A)}(\kappa * \lambda)$}.
Therefore $\psi^{+}_{\mathcal{I}(\mathcal{I}(A))}(\lambda * \mu) \leq$ max { $\psi^{+}_{\mathcal{I}(A)}(\kappa * \mu),\ \psi^{+}_{\mathcal{I}(A)}(\kappa * \lambda)$}.

f) Now $\psi^{-}_{\mathcal{I}(\mathcal{I}(A))}(\lambda * \mu)$= min $\psi^{-}_{\mathcal{I}(A)}(b * c)$ = min (min $\psi^{-}_{A}(\lambda * \mu)$) = min ($\psi^{-}_{A}(b * c)$)
$\geq$ min { min { $\psi^{-}_{A}(a * c),\ \psi^{-}_{A}(a * b)$}}
= min { min $\psi^{-}_{A}(a * c)$, min $\psi^{-}_{A}(a * b)$}
= min { $\psi^{-}_{\mathcal{I}(A)}(\kappa * \mu),\ \psi^{-}_{\mathcal{I}(A)}(\kappa * \lambda)$}.
Therefore $\psi^{-}_{\mathcal{I}(\mathcal{I}(A))}(\lambda * \mu) \geq$ min { $\psi^{-}_{\mathcal{I}(A)}(\kappa * \mu),\ \psi^{-}_{\mathcal{I}(A)}(\kappa * \lambda)$}.

Therefore $\mathcal{I}(\mathcal{I}(A)) = \mathcal{I}(A)$ is a bipolar intuitionistic fuzzy α-ideal of Γ.

Theorem: 3.3
If A and B are bipolar intuitionistic fuzzy α-ideals of Γ, then $\mathcal{I}(A \cap B) = \mathcal{I}(A) \cap \mathcal{I}(B)$ is a bipolar intuitionistic fuzzy α-ideal of Γ.

Proof: Given A and B are bipolar intuitionistic fuzzy α-ideals of Γ.
Consider $0,\ \kappa,\ \lambda,\ \mu \in A \cap B$ then $0, \kappa,\ \lambda,\ \mu \in A$ and $0, \kappa,\ \lambda,\ \mu \in B$.

a) Now $\chi^{+}_{\mathcal{I}(A \cap B)}(0)$ = min $\chi^{+}_{A \cap B}(\kappa)$ = min (min ($\chi^{+}_{A}(\kappa),\ \chi^{+}_{B}(\kappa)$))
= min (min $\chi^{+}_{A}(\kappa)$, min $\chi^{+}_{B}(\kappa)$) = min ($\chi^{+}_{A}(a),\ \chi^{+}_{B}(a)$)
$\geq$ min (min $\chi^{+}_{A}(a)$, min $\chi^{+}_{B}(a)$)
= min ($\chi^{+}_{\mathcal{I}(A)}(\kappa),\ \chi^{+}_{\mathcal{I}(B)}(\kappa)$) = $\chi^{+}_{\mathcal{I}(A) \cap \mathcal{I}(B)}(\kappa)$.
Therefore $\chi^{+}_{\mathcal{I}(A \cap B)}(0) \geq \chi^{+}_{\mathcal{I}(A) \cap \mathcal{I}(B)}(\kappa)$.
Now $\chi^{-}_{\mathcal{I}(A \cap B)}(0)$ = max $\chi^{-}_{A \cap B}(\kappa)$ = max (max ($\chi^{-}_{A}(\kappa),\ \chi^{-}_{B}(\kappa)$))
= max (max $\chi^{-}_{A}(\kappa)$, max $\chi^{-}_{B}(\kappa)$) = max ($\chi^{-}_{A}(a),\ \chi^{-}_{B}(a)$)
$\leq$ max (max $\chi^{-}_{A}(a)$, max $\chi^{-}_{B}(a)$)
= max ($\chi^{-}_{\mathcal{I}(A)}(\kappa),\ \chi^{-}_{\mathcal{I}(B)}(\kappa)$) = $\chi^{-}_{\mathcal{I}(A) \cap \mathcal{I}(B)}(\kappa)$.
Therefore $\chi^{-}_{\mathcal{I}(A \cap B)}(0) \leq \chi^{-}_{\mathcal{I}(A) \cap \mathcal{I}(B)}(\kappa)$.

b) Now $\chi^{+}_{\mathcal{I}(A \cap B)}(\lambda * \mu)$ = min $\chi^{+}_{A \cap B}(b * c)$ = min (min ($\chi^{+}_{A}(b * c),\ \chi^{+}_{B}(b * c)$))
= min (min $\chi^{+}_{A}(b * c)$, min $\chi^{+}_{B}(b * c)$)
$\geq$ min (min { min { $\chi^{+}_{A}(a * c),\ \chi^{+}_{A}(a * b)$}} ,
min { min { $\chi^{+}_{B}(a * c),\ \chi^{+}_{B}(a * b)$}})

CAHIERS MAGELLANES-NS
Volume 06 Issue 2
2024
ISSN:1624-1940
DOI 10.6084/m9.figshare.26310216
http://magellanes.com/

$= \min \{ \min (\min \{\chi_{A}^{+}(a*c),\ \chi_{A}^{+}(a*b)\}),\ \min (\min \{\chi_{B}^{+}(a*c),\ \chi_{B}^{+}(a*b)\})\}$

$= \min \{ \min (\min \{\chi_{A}^{+}(a*c),\ \chi_{B}^{+}(a*c)\}),\ \min (\min \{\chi_{A}^{+}(a*b),\ \chi_{B}^{+}(a*b)\})\}$

$= \min \{ \min (\min \chi_{A}^{+}(a*c), \min \chi_{B}^{+}(a*c)),\ \min (\min \chi_{A}^{+}(a*b),\ \min \chi_{B}^{+}(a*b))\}$

$= \min \{ \min (\chi_{\mathcal{J}(A)}^{+}(\kappa*\mu),\ \chi_{\mathcal{J}(B)}^{+}(\kappa*\mu)),\ \min (\chi_{\mathcal{J}(A)}^{+}(\kappa*\lambda),\ \chi_{\mathcal{J}(B)}^{+}(\kappa*\lambda))\}$

$= \min \{ \chi_{\mathcal{J}(A)\cap\mathcal{J}(B)}^{+}(\kappa*\mu),\ \chi_{\mathcal{J}(A)\cap\mathcal{J}(B)}^{+}(\kappa*\lambda)\}.$

Therefore $\chi_{\mathcal{J}(A\cap B)}^{+}(\lambda*\mu) \geq \min \{ \chi_{\mathcal{J}(A)\cap\mathcal{J}(B)}^{+}(\kappa*\mu),\ \chi_{\mathcal{J}(A)\cap\mathcal{J}(B)}^{+}(\kappa*\lambda)\}.$

c) Now $\chi_{\mathcal{J}(A\cap B)}^{-}(\lambda*\mu) = \max\ \chi_{A\cap B}^{-}(b*c) = \max (\max (\chi_{A}^{-}(b*c),\ \chi_{B}^{-}(b*c)))$

$= \max (\max \chi_{A}^{-}(b*c),\ \max \chi_{B}^{-}(b*c))$

$\leq \max (\max \{ \max \{ \chi_{A}^{-}(a*c),\ \chi_{A}^{-}(a*b)\}\},$
$\max \{ \max \{ \chi_{B}^{-}(a*c),\ \chi_{B}^{-}(a*b)\}\})$

$= \max \{ \max (\max \{ \chi_{A}^{-}(a*c),\ \chi_{A}^{-}(a*b)\}),\ \max (\max \{ \chi_{B}^{-}(a*c),\ \chi_{B}^{-}(a*b)\})\}$

$= \max \{ \max (\max \{ \chi_{A}^{-}(a*c),\ \chi_{B}^{-}(a*c)\}),\ \max (\max \{ \chi_{A}^{-}(a*b),\ \chi_{B}^{-}(a*b)\})\}$

$= \max \{ \max (\max \chi_{A}^{-}(a*c),\ \max \chi_{B}^{-}(a*c)),\ \max (\max \chi_{A}^{-}(a*b), \max \chi_{B}^{-}(a*b))\}$

$= \max \{ \max (\chi_{\mathcal{J}(A)}^{-}(\kappa*\mu),\ \chi_{\mathcal{J}(B)}^{-}(\kappa*\mu)),\ \max (\chi_{\mathcal{J}(A)}^{-}(\kappa*\lambda),\ \chi_{\mathcal{J}(B)}^{-}(\kappa*\lambda))\}$

$= \max \{ \chi_{\mathcal{J}(A)\cap\mathcal{J}(B)}^{-}(\kappa*\mu),\ \chi_{\mathcal{J}(A)\cap\mathcal{J}(B)}^{-}(\kappa*\lambda)\}.$

Therefore $\chi_{\mathcal{J}(A\cap B)}^{-}(\lambda*\mu) \leq \max \{ \chi_{\mathcal{J}(A)\cap\mathcal{J}(B)}^{-}(\kappa*\mu),\ \chi_{\mathcal{J}(A)\cap\mathcal{J}(B)}^{-}(\kappa*\lambda)\}.$

d) Now $\psi_{\mathcal{J}(A\cap B)}^{+}(0) = \max\ \psi_{A\cap B}^{+}(\kappa) = \max (\max (\psi_{A}^{+}(\kappa),\ \psi_{B}^{+}(\kappa)))$

$= \max (\max\ \psi_{A}^{+}(\kappa),\ \max\ \psi_{B}^{+}(\kappa)) = \max (\psi_{A}^{+}(a),\ \psi_{B}^{+}(a))$

$\leq \max (\max\ \psi_{A}^{+}(a),\ \max\ \psi_{B}^{+}(a))$

$= \max (\psi_{\mathcal{J}(A)}^{+}(\kappa),\ \psi_{\mathcal{J}(B)}^{+}(\kappa)) = \psi_{\mathcal{J}(A)\cap\mathcal{J}(B)}^{+}(\kappa).$

Therefore $\psi_{\mathcal{J}(A\cap B)}^{+}(0) \leq \psi_{\mathcal{J}(A)\cap\mathcal{J}(B)}^{+}(\kappa).$

Now $\psi_{\mathcal{J}(A\cap B)}^{-}(0) = \min\ \psi_{A\cap B}^{-}(\kappa) = \min (\min (\psi_{A}^{-}(\kappa),\ \psi_{B}^{-}(\kappa)))$

$= \min (\min \psi_{A}^{-}(\kappa),\ \min \psi_{B}^{-}(\kappa)) = \min (\psi_{A}^{-}(a),\ \psi_{B}^{-}(a))$

$\geq \min (\min \psi_{A}^{-}(a),\ \min \psi_{B}^{-}(a))$

$= \min (\psi_{\mathcal{J}(A)}^{-}(\kappa),\ \psi_{\mathcal{J}(B)}^{-}(\kappa)) = \psi_{\mathcal{J}(A)\cap\mathcal{J}(B)}^{-}(\kappa).$

Therefore $\psi_{\mathcal{J}(A\cap B)}^{-}(0) \geq \psi_{\mathcal{J}(A)\cap\mathcal{J}(B)}^{-}(\kappa).$

e) Now $\psi_{\mathcal{J}(A\cap B)}^{+}(\lambda*\mu) = \max\ \psi_{A\cap B}^{+}(b*c) = \max (\max (\psi_{A}^{+}(b*c),\ \psi_{B}^{+}(b*c)))$

$= \max (\max \psi_{A}^{+}(b*c),\ \max \psi_{B}^{+}(b*c))$

$\leq \max (\max \{ \max \{ \psi_{A}^{+}(a*c),\ \psi_{A}^{+}(a*b)\}\},$
$\max \{ \max \{ \psi_{B}^{+}(a*c),\ \psi_{B}^{+}(a*b)\}\})$

$= \max \{ \max (\max \{ \psi_{A}^{+}(a*c),\ \psi_{A}^{+}(a*b)\}),\ \max (\max \{ \psi_{B}^{+}(a*c),\ \psi_{B}^{+}(a*b)\})\}$

$= \max \{ \max (\max \{ \psi_{A}^{+}(a*c), \psi_{B}^{+}(a*c)\}),\ \max (\max \{\psi_{A}^{+}(a*b),\ \psi_{B}^{+}(a*b)\})\}$

$= \max \{ \max (\max \psi_{A}^{+}(a*c),\ \max \psi_{B}^{+}(a*c)),\ \max (\max \psi_{A}^{+}(a*b),\ \max \psi_{B}^{+}(a*b))\}$

$= \max \{ \max (\psi_{\mathcal{J}(A)}^{+}(\kappa*\mu),\ \psi_{\mathcal{J}(B)}^{+}(\kappa*\mu)),\ \max (\psi_{\mathcal{J}(A)}^{+}(\kappa*\lambda),\ \psi_{\mathcal{J}(B)}^{+}(\kappa*\lambda))\}$

$= \max \{ \psi_{\mathcal{J}(A)\cap\mathcal{J}(B)}^{+}(\kappa*\mu),\ \psi_{\mathcal{J}(A)\cap\mathcal{J}(B)}^{+}(\kappa*\lambda)\}$

Therefore $\psi_{\mathcal{J}(A\cap B)}^{+}(\lambda*\mu) \leq \max \{ \psi_{\mathcal{J}(A)\cap\mathcal{J}(B)}^{+}(\kappa*\mu),\ \psi_{\mathcal{J}(A)\cap\mathcal{J}(B)}^{+}(\kappa*\lambda)\}.$

f) Now $\psi_{\mathcal{J}(A\cap B)}^{-}(\lambda*\mu) = \min\ \psi_{A\cap B}^{-}(b*c)$

$= \min (\min (\psi_{A}^{-}(b*c),\ \psi_{B}^{-}(b*c)))$

$= \min (\min \psi_{A}^{-}(b*c),\ \min \psi_{B}^{-}(b*c))$

$\geq \min (\min \{ \min \{ \psi_{A}^{-}(a*c),\ \psi_{A}^{-}(a*b) \}\},$
$\min \{ \min \{ \psi_{B}^{-}(a*c),\ \psi_{B}^{-}(a*b) \}\})$

$= \min \{ \min (\min \{ \psi_{A}^{-}(a*c),\ \psi_{A}^{-}(a*b)\}), \min (\min \{ \psi_{B}^{-}(a*c),\ \psi_{B}^{-}(a*b) \}) \}$

ISSN:1624-1940
DOI 10.6084/m9.figshare.26310216
http://magellanes.com/

$= \min \{ \min (\min \{ \psi^-_{\mathrm{A}}(a * c),\ \psi^-_{\mathrm{B}}(a * c)\}),\ \min (\min \{ \psi^-_{\mathrm{A}}(a * b)\ ,\ \psi^-_{\mathrm{B}}(a * b) \}) \}$
$= \min \{ \min (\min \psi^-_{\mathrm{A}}(a * c),\ \min \psi^-_{\mathrm{B}}(a * c)),\ \min (\min \psi^-_{\mathrm{A}}(a * b),\ \min \psi^-_{\mathrm{B}}(a * b)) \}$
$= \min \{ \min (\psi^-_{\mathcal{J}(\mathrm{A})}(\kappa * \mu),\ \psi^-_{\mathcal{J}(\mathrm{B})}(\kappa * \mu)),\ \min (\psi^-_{\mathcal{J}(\mathrm{A})}(\kappa * \lambda),\ \psi^-_{\mathcal{J}(\mathrm{B})}(\kappa * \lambda)) \}$
$= \min \{ \psi^-_{\mathcal{J}(\mathrm{A})\cap \mathcal{J}(\mathrm{B})}(\kappa * \mu),\ \ \psi^-_{\mathcal{J}(\mathrm{A})\cap \mathcal{J}(\mathrm{B})}(\kappa * \lambda)\}$.

Therefore $\psi^-_{\mathcal{J}(\mathrm{A} \cap \mathrm{B})}(\lambda * \mu) \geq \min \{ \psi^-_{\mathcal{J}(\mathrm{A})\cap \mathcal{J}(\mathrm{B})}(\kappa * \mu),\ \ \psi^-_{\mathcal{J}(\mathrm{A})\cap \mathcal{J}(\mathrm{B})}(\kappa * \lambda)\}$.
Therefore $\mathcal{J}(\mathrm{A} \cap \mathrm{B}) = \mathcal{J}(\mathrm{A}) \cap \mathcal{J}(\mathrm{B})$ is a bipolar intuitionistic fuzzy α-ideal of Γ.

Theorem: 3.4
If A is a bipolar intuitionistic fuzzy α-ideal of Γ, then $\square(\mathcal{J}(\mathrm{A})) = \mathcal{J}(\square(\mathrm{A}))$ is a bipolar intuitionistic fuzzy α-ideal of Γ.

Proof: Given A is a bipolar intuitionistic fuzzy α-ideal of Γ.
Consider $0, \kappa, \lambda, \mu \in \mathrm{A}$.

a) Now $\chi^+_{\square(\mathcal{J}(\mathrm{A}))}(0) = \chi^+_{\mathcal{J}(\mathrm{A})}(0) = \min\ \chi^+_{\mathrm{A}}(\kappa) = \chi^+_{\mathrm{A}}(a)$
$\geq \min \chi^+_{\mathrm{A}}(a) = \min\ \chi^+_{\square(\mathrm{A})}(a) = \chi^+_{\mathcal{J}(\square(\mathrm{A}))}(\kappa)$.
Therefore $\chi^+_{\square(\mathcal{J}(\mathrm{A}))}(0) \geq \chi^+_{\mathcal{J}(\square(\mathrm{A}))}(\kappa)$.
Now $\chi^-_{\square(\mathcal{J}(\mathrm{A}))}(0) = \chi^-_{\mathcal{J}(\mathrm{A})}(0) = \max\ \chi^-_{\mathrm{A}}(\kappa) = \chi^-_{\mathrm{A}}(a)$
$\leq \max \chi^-_{\mathrm{A}}(a) = \max\ \chi^-_{\square(\mathrm{A})}(a) = \chi^-_{\mathcal{J}(\square(\mathrm{A}))}(\kappa)$.
Therefore $\chi^-_{\square(\mathcal{J}(\mathrm{A}))}(0) \leq \chi^-_{\mathcal{J}(\square(\mathrm{A}))}(\kappa)$.

b) Now $\chi^+_{\square(\mathcal{J}(\mathrm{A}))}(\lambda * \mu) = \chi^+_{\mathcal{J}(\mathrm{A})}(\lambda * \mu) = \min\ \chi^+_{\mathrm{A}}(b * c)$
$\geq \min \{ \min \{ \chi^+_{\mathrm{A}}(a * c),\ \chi^+_{\mathrm{A}}(a * b)\}\}$
$= \min \{ \min \{ \chi^+_{\square(\mathrm{A})}(a * c),\ \chi^+_{\square(\mathrm{A})}(a * b)\}\}$
$= \min \{ \min\ \chi^+_{\square(\mathrm{A})}(a * c), \min\ \chi^+_{\square(\mathrm{A})}(a * b)\}$
$= \min \{ \chi^+_{\mathcal{J}(\square(\mathrm{A}))}(\kappa * \mu),\ \ \chi^+_{\mathcal{J}(\square(\mathrm{A}))}(\kappa * \lambda)\}$.
Therefore $\chi^+_{\square(\mathcal{J}(\mathrm{A}))}(\lambda * \mu) \geq \min \{ \chi^+_{\mathcal{J}(\square(\mathrm{A}))}(\kappa * \mu),\ \chi^+_{\mathcal{J}(\square(\mathrm{A}))}(\kappa * \lambda)\}$.

c) Now $\chi^-_{\square(\mathcal{J}(\mathrm{A}))}(\lambda * \mu) = \chi^-_{\mathcal{J}(\mathrm{A})}(\lambda * \mu) = \max \chi^-_{\mathrm{A}}(b * c)$
$\leq \max \{ \max \{ \chi^-_{\mathrm{A}}(a * c),\ \chi^-_{\mathrm{A}}(a * b)\}\}$
$= \max \{ \max \{ \chi^-_{\square(\mathrm{A})}(a * c),\ \chi^-_{\square(\mathrm{A})}(a * b)\}\}$
$= \max \{ \max \chi^-_{\square(\mathrm{A})}(a * c),\ \max \chi^-_{\square(\mathrm{A})}(a * b)\}$
$= \max \{ \chi^-_{\mathcal{J}(\square(\mathrm{A}))}(\kappa * \mu),\ \ \chi^-_{\mathcal{J}(\square(\mathrm{A}))}(\kappa * \lambda)\}$.
Therefore $\chi^-_{\square(\mathcal{J}(\mathrm{A}))}(\lambda * \mu) \leq \max \{ \chi^-_{\mathcal{J}(\square(\mathrm{A}))}(\kappa * \mu),\ \ \chi^-_{\mathcal{J}(\square(\mathrm{A}))}(\kappa * \lambda)\}$.

d) Now $\psi^+_{\square(\mathcal{J}(\mathrm{A}))}(0) = 1 - \chi^+_{\mathcal{J}(\mathrm{A})}(0) = \max (1 - \chi^+_{\mathrm{A}}(\kappa)) = 1 - \chi^+_{\mathrm{A}}(a)$
$\leq \max (1 - \chi^+_{\mathrm{A}}(a)) = \max (1 - \chi^+_{\square(\mathrm{A})}(a))$
$= \max \psi^+_{\square(\mathrm{A})}(a) = \psi^+_{\mathcal{J}(\square(\mathrm{A}))}(\kappa)$.
Therefore $\psi^+_{\square(\mathcal{J}(\mathrm{A}))}(0) \leq \psi^+_{\mathcal{J}(\square(\mathrm{A}))}(\kappa)$.
Now $\psi^-_{\square(\mathcal{J}(\mathrm{A}))}(0) = 1 - \chi^-_{\mathcal{J}(\mathrm{A})}(0) = \min (1 - \chi^-_{\mathrm{A}}(\kappa)) = 1 - \chi^-_{\mathrm{A}}(a)$
$\geq \min (1 - \chi^-_{\mathrm{A}}(a)) = \min (1 - \chi^-_{\square(\mathrm{A})}(a))$
$= \min \psi^-_{\square(\mathrm{A})}(a) = \psi^-_{\mathcal{J}(\square(\mathrm{A}))}(\kappa)$.
Therefore $\psi^-_{\square(\mathcal{J}(\mathrm{A}))}(0) \geq \psi^-_{\mathcal{J}(\square(\mathrm{A}))}(\kappa)$.

e) Now $\psi^+_{\square(\mathcal{J}(\mathrm{A}))}(\lambda * \mu) = 1 - \chi^+_{\mathcal{J}(\mathrm{A})}(\lambda * \mu) = \max (1 - \chi^+_{\mathrm{A}}(b * c))$
$\leq \max \{ \max \{ 1 - \chi^+_{\mathrm{A}}(a * c),\ 1 - \chi^+_{\mathrm{A}}(a * b)\}\}$

CAHIERS MAGELLANES-NS
Volume 06 Issue 2
2024
ISSN:1624-1940
DOI 10.6084/m9.figshare.26310216
http://magellanes.com/

$= \max \{ \max \{ 1 - \chi^{+}_{\square(A)}(a * c),\ 1 - \chi^{+}_{\square(A)}(a * b)\}\}$
$= \max \{ \max (1 - \chi^{+}_{\square(A)}(a * c)), \max (1 - \chi^{+}_{\square(A)}(a * b))\}$
$= \max \{ \max \psi^{+}_{\square(A)}(a * c),\ \max \psi^{+}_{\square(A)}(a * b)\}$
$= \max \{ \psi^{+}_{\mathcal{J}(\square(A))}(\kappa * \mu),\ \psi^{+}_{\mathcal{J}(\square(A))}(\kappa * \lambda)\}.$

Therefore $\psi^{+}_{\square(\mathcal{J}(A))}(\lambda * \mu) \leq \max \{ \psi^{+}_{\mathcal{J}(\square(A))}(\kappa * \mu),\ \psi^{+}_{\mathcal{J}(\square(A))}(\kappa * \lambda)\}.$

f) Now $\psi^{-}_{\square(\mathcal{J}(A))}(\lambda * \mu) = 1 - \chi^{-}_{\mathcal{J}(A)}(\lambda * \mu) = \min (1 - \chi^{-}_{A}(b * c))$
$\geq \min \{ \min \{ 1 - \chi^{-}_{A}(a * c),\ 1 - \chi^{-}_{A}(a * b)\}\}$
$= \min \{ \min \{ 1 - \chi^{-}_{\square(A)}(a * c),\ 1 - \chi^{-}_{\square(A)}(a * b)\}\}$
$= \min \{ \min (1 - \chi^{-}_{\square(A)}(a * c)),\ \min (1 - \chi^{-}_{\square(A)}(a * b))\}$
$= \min \{ \min \psi^{-}_{\square(A)}(a * c),\ \min \psi^{-}_{\square(A)}(a * b)\}$
$= \min \{ \psi^{-}_{\mathcal{J}(\square(A))}(\kappa * \mu),\ \psi^{-}_{\mathcal{J}(\square(A))}(\kappa * \lambda)\}.$

Therefore $\psi^{-}_{\square(\mathcal{J}(A))}(\lambda * \mu) \geq \min \{ \psi^{-}_{\mathcal{J}(\square(A))}(\kappa * \mu),\ \psi^{-}_{\mathcal{J}(\square(A))}(\kappa * \lambda)\}.$
Therefore $\square(\mathcal{J}(A)) = \mathcal{J}(\square(A))$ is a bipolar intuitionistic fuzzy α-ideal of Γ.

Theorem: 3.5
If A is a bipolar intuitionistic fuzzy α-ideal of Γ, then $\Diamond(\mathcal{J}(A)) = \mathcal{J}(\Diamond(A))$ is a bipolar intuitionistic fuzzy α-ideal of Γ.

Proof: Given A is a bipolar intuitionistic fuzzy α-ideal of Γ.
Consider $0, \kappa, \lambda, \mu \in A$.

a) Now $\chi^{+}_{\Diamond(\mathcal{J}(A))}(0) = 1 - \psi^{+}_{\mathcal{J}(A)}(0) = \min (1 - \psi^{+}_{A}(\kappa)) = 1 - \psi^{+}_{A}(a)$
$\geq \min (1 - \psi^{+}_{A}(a)) = \min (1 - \psi^{+}_{\Diamond(A)}(a))$
$= \min \chi^{+}_{\Diamond(A)}(a) = \chi^{+}_{\mathcal{J}(\Diamond(A))}(\kappa).$

Therefore $\chi^{+}_{\Diamond(\mathcal{J}(A))}(0) \geq \chi^{+}_{\mathcal{J}(\Diamond(A))}(\kappa).$

Now $\chi^{-}_{\Diamond(\mathcal{J}(A))}(0) = 1 - \psi^{-}_{\mathcal{J}(A)}(0) = \max (1 - \psi^{-}_{A}(\kappa)) = 1 - \psi^{-}_{A}(a)$
$\leq \max (1 - \psi^{-}_{A}(a)) = \max (1 - \psi^{-}_{\Diamond(A)}(a))$
$= \max \chi^{-}_{\Diamond(A))}(a) = \chi^{-}_{\mathcal{J}(\Diamond(A))}(\kappa).$

Therefore $\chi^{-}_{\Diamond(\mathcal{J}(A))}(0) \leq \chi^{-}_{\mathcal{J}(\Diamond(A))}(\kappa).$

b) Now $\chi^{+}_{\Diamond(\mathcal{J}(A))}(\lambda * \mu) = 1 - \psi^{+}_{\mathcal{J}(A)}(\lambda * \mu) = \min (1 - \psi^{+}_{A}(b * c))$
$\geq \min \{ \min \{ 1 - \psi^{+}_{A}(a * c),\ 1 - \psi^{+}_{A}(a * b)\}\}$
$= \min \{ \min \{ 1 - \psi^{+}_{\Diamond(A)}(a * c),\ 1 - \psi^{+}_{\Diamond(A)}(a * b)\}\}$
$= \min \{ \min (1 - \psi^{+}_{\Diamond(A)}(a * c)),\ \min (1 - \psi^{+}_{\Diamond(A)}(a * b))\}$
$= \min \{ \min \chi^{+}_{\Diamond(A)}(a * c),\ \min \chi^{+}_{\Diamond(A)}(a * b)\}$
$= \min \{ \chi^{+}_{\mathcal{J}(\Diamond(A))}(\kappa * \mu),\ \chi^{+}_{\mathcal{J}(\Diamond(A))}(\kappa * \lambda)\}.$

Therefore $\chi^{+}_{\Diamond(\mathcal{J}(A))}(\lambda * \mu) \geq \min \{ \chi^{+}_{\mathcal{J}(\Diamond(A))}(\kappa * \mu),\ \chi^{+}_{\mathcal{J}(\Diamond(A))}(\kappa * \lambda)\}.$

c) Now $\chi^{-}_{\Diamond(\mathcal{J}(A))}(\lambda * \mu) = 1 - \psi^{-}_{\mathcal{J}(A)}(\lambda * \mu) = \max (1 - \psi^{-}_{A}(b * c))$
$\leq \max \{ \max \{ 1 - \psi^{-}_{A}(a * c),\ 1 - \psi^{-}_{A}(a * b) \}\}$
$= \max \{ \max \{ 1 - \psi^{-}_{\Diamond(A)}(a * c),\ 1 - \psi^{-}_{\Diamond(A)}(a * b) \}\}$
$= \max \{ \max (1 - \psi^{-}_{\Diamond(A)}(a * c)) , \max (1 - \psi^{-}_{\Diamond(A)}(a * b))\}$
$= \max \{ \max \chi^{-}_{\Diamond(A))}(a * c) ,\ \max \chi^{-}_{\Diamond(A))}(a * b)\}$
$= \max \{\chi^{-}_{\mathcal{J}(\Diamond(A))}(\kappa * \mu) ,\ \chi^{-}_{\mathcal{J}(\Diamond(A))}(\kappa * \lambda)\}.$

ISSN:1624-1940
DOI 10.6084/m9.figshare.26310216
http://magellanes.com/

Therefore $\chi^{-}_{\Diamond(\mathcal{J}(\mathrm{A}))}(\lambda * \mu) \leq \max \{ \chi^{-}_{\mathcal{J}(\Diamond(\mathrm{A}))}(\kappa * \mu), \ \chi^{-}_{\mathcal{J}(\Diamond(\mathrm{A}))}(\kappa * \lambda)\}$.

d) Now $\psi^{+}_{\Diamond(\mathcal{J}(\mathrm{A}))}(0) = \psi^{+}_{\mathcal{J}(\mathrm{A})}(0) = \max \psi^{+}_{\mathrm{A}}(\kappa) = \psi^{+}_{\mathrm{A}}(a)$
$\leq \max \psi^{+}_{\mathrm{A}}(a) = \max \psi^{+}_{\Diamond(\mathrm{A})}(a) = \psi^{+}_{\mathcal{J}(\Diamond(\mathrm{A}))}(\kappa)$.

Therefore $\psi^{+}_{\Diamond(\mathcal{J}(\mathrm{A}))}(0) \leq \psi^{+}_{\mathcal{J}(\Diamond(\mathrm{A}))}(\kappa)$.

Now $\psi^{-}_{\Diamond(\mathcal{J}(\mathrm{A}))}(0) = \psi^{-}_{\mathcal{J}(\mathrm{A})}(0) = \min \psi^{-}_{\mathrm{A}}(\kappa) = \psi^{-}_{\mathrm{A}}(a)$
$\geq \min \psi^{-}_{\mathrm{A}}(a) = \min \psi^{-}_{\Diamond(\mathrm{A})}(a) = \psi^{-}_{\mathcal{J}(\Diamond(\mathrm{A}))}(\kappa)$.

Therefore $\psi^{-}_{\Diamond(\mathcal{J}(\mathrm{A}))}(0) \geq \psi^{-}_{\mathcal{J}(\Diamond(\mathrm{A}))}(\kappa)$.

e) Now $\psi^{+}_{\Diamond(\mathcal{J}(\mathrm{A}))}(\lambda * \mu) = \psi^{+}_{\mathcal{J}(\mathrm{A})}(\lambda * \mu) = \max \psi^{+}_{\mathrm{A}}(b * c)$
$\leq \max \{ \max \{ \psi^{+}_{\mathrm{A}}(a * c), \ \psi^{+}_{\mathrm{A}}(a * b)\}\}$
$= \max \{ \max \{ \psi^{+}_{\Diamond(\mathrm{A})}(a * c), \ \psi^{+}_{\Diamond(\mathrm{A})}(a * b)\}\}$
$= \max \{ \max \psi^{+}_{\Diamond(\mathrm{A})}(a * c), \ \max \psi^{+}_{\Diamond(\mathrm{A})}(a * b)\}$
$= \max \{ \psi^{+}_{\mathcal{J}(\Diamond(\mathrm{A}))}(\kappa * \mu), \ \psi^{+}_{\mathcal{J}(\Diamond(\mathrm{A}))}(\kappa * \lambda)\}$.

Therefore $\psi^{+}_{\Diamond(\mathcal{J}(\mathrm{A}))}(\lambda * \mu) \leq \max \{ \psi^{+}_{\mathcal{J}(\Diamond(\mathrm{A}))}(\kappa * \mu), \ \psi^{+}_{\mathcal{J}(\Diamond(\mathrm{A}))}(\kappa * \lambda)\}$.

f) Now $\psi^{-}_{\Diamond(\mathcal{J}(\mathrm{A}))}(\lambda * \mu) = \psi^{-}_{\mathcal{J}(\mathrm{A})}(\lambda * \mu) = \min \psi^{-}_{\mathrm{A}}(b * c)$
$\geq \min \{ \min \{ \psi^{-}_{\mathrm{A}}(a * c), \ \psi^{-}_{\mathrm{A}}(a * b) \}\}$
$= \min \{ \min \{ \psi^{-}_{\Diamond(\mathrm{A})}(a * c), \ \psi^{-}_{\Diamond(\mathrm{A})}(a * b) \}\}$
$= \min \{ \min \psi^{-}_{\Diamond(\mathrm{A})}(a * c), \ \min \psi^{-}_{\Diamond(\mathrm{A})}(a * b)\}$
$= \min \{ \psi^{-}_{\mathcal{J}(\Diamond(\mathrm{A}))}(\kappa * \mu), \ \psi^{-}_{\mathcal{J}(\Diamond(\mathrm{A}))}(\kappa * \lambda)\}$.

Therefore $\psi^{-}_{\Diamond(\mathcal{J}(\mathrm{A}))}(\lambda * \mu) \geq \min \{ \psi^{-}_{\mathcal{J}(\Diamond(\mathrm{A}))}(\kappa * \mu), \ \psi^{-}_{\mathcal{J}(\Diamond(\mathrm{A}))}(\kappa * \lambda)\}$.

Therefore $\Diamond(\mathcal{J}(\mathrm{A})) = \mathcal{J}(\Diamond(\mathrm{A}))$ is a bipolar intuitionistic fuzzy α-ideal of Γ.

4. Conclusion

This article uses the new algebraic structure of BP-algebra, which is a bipolar intuitionistic fuzzy α-ideal, through the inner operator. The study's objective is carried out. The discussion focuses on the relationship between the operations of topological operators on bipolar intuitionistic fuzzy α-ideal. We think that other algebraic systems can also benefit from our concepts.

5. References

1. S.Abdullah and M.M.M. Aslam, Bipolar fuzzy ideals in LA-semigroups, World Appl. Sci. J., 17.12 (2012), 1769-1782.
2. S.S.Ahn and J.S.Han, On BP-algebra, Hacettepe Journal of Mathematics and Statistics, 42 (2013), 551-557.
3. K.Chakrabarthy, Biswas R.Nanda, A note on union and intersection of intuitionistic fuzzy sets, Notes on intuitionistic fuzzy sets, 3(4), 1997.
4. K.J.Lee, Bipolar-valued fuzzy sets and their basic operations, Proc. Int. Conf., Bangkok, Thailand, 2007, 307-317.
5. Osama Rashad EI-Gendy, Bipolar fuzzy α-ideal of BP –algebra, American Journal of Mathematics and Statistics 2020, 10(2): 33-37.
6. M.Palanivelrajan and S.Nandakumar, Some operations of intuitionistic fuzzy primary and semiprimary ideal, Asian journal of algebra 5, No.2 (2012), 44-49.
7. A. Rajeshkumar, Fuzzy Algebra: Volume I (Publication division,University of Delhi).
8. L.A.Zadeh, Fuzzy sets, Information Control, 8 (1965), 338-353.
9. W.R.Zhang, Bipolar fuzzy sets, Part I, Proc. of FUZZ-IEEE, 2 (1998), 835-840.