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A NEW CLASS OF INTEGRALS INVOLVING K-GENERALIZED MITTAG-LEFFLER FUNCTION

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ABSTRACT:
In this paper we established integral formulas involving K-generalized Mittag-Leffler function and result is transform in terms of generalized beta and gamma function. Some interesting transforms of our main results are also considered. The results are derived with the help of an interesting integral due to Lavoie and troitter

Key words: Mittag-Leffler function, Beta and gamma functions, generalized Mittag-Leffler function, k-Beta and k-Gamma function, pochhammer symbols and lavoie-troitter integral.

1. Introduction and preliminaries:

In recent years, many integral formulae involving a variety of special functions have been developed by various authors [2, 3, 4] for a very recent work, see also [1].The Mittag-Leffler function is being studied due to its wide area of applications like applications in solving fractional differential and integral equations, etc. see ([6],[7],[8],[9]).In 1903, the Swedish mathematician Gosta Mittag-Leffler[5] introduced the function Eα(z) known as one-parameter Mittag-Leffler function defined by

$$E_l(z) = \sum_{n=0}^{\infty} \frac{z^n}{\Gamma(nl+1)} \tag{1}$$

Where, z is a complex variable and Γ (.) is a gamma function, The Mittag-Leffler Function is a direct generalization of the exponential function to which it reduces for l = 1 and for $0<l<1$it interpolates between the pure exponential and a hypergeometric function 1/1−z.
Its generalization was given by Wiman [10]:

$$E_{l,m}(z) = \sum_{n=0}^{\infty} \frac{z^n}{\Gamma(nl+m)} \quad ,z,l,m\epsilon\mathbb{C}\,,\Re(l)>0\,,\Re(m)>0 \tag{2}$$

In 1971, Prabhakar extended these above definitions ofMLf in the following form [11]:

$$E_{l,m}^{\rho}\,(z) = \sum_{n=0}^{\infty} \frac{(\rho)_n z^n}{\Gamma(nl+m)n!} \quad ,z,l,m,\rho\epsilon\mathbb{C}\,,\Re(l)>0\,,\Re(m)>0 \tag{3}$$

where, $(\rho)_n = \dfrac{\Gamma(\rho+n)}{\Gamma(\rho)}$ is the well − known Pochhammer symbol.

Recently generalization of Prabhakar's MLf was done in 2007 by Shukla and Prajapati [12]:

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$$E_{l,m}^{\rho,\sigma}(z)=\sum_{n=0}^{\infty}\frac{(\rho)_{n\sigma}\ \ z^n}{\Gamma(nl+m)n!}\quad ,z,l,m,\rho\epsilon\mathbb{C}\,,\Re(l)>0\,,\Re(m)>0\,,$$

$\Re(\rho)>0\ \ \sigma\epsilon(0,1)\cup\mathbb{N}$ (4)

Where $(\rho)_{n\sigma}\ =\frac{\Gamma(\rho+n\sigma)}{\Gamma(\rho)}$ denotes generalized pochhammer symbol.

In 2012, a new concept to extend the definition ofMLf was developed by G. A. Dorrego et al. using the k-Gamma function. In the literature, it is defined and Represented as follows [13]:

$$E_{k,l,m}^{\rho}(z)=\sum_{n=0}^{\infty}\frac{(\rho)_{n,k}z^n}{\Gamma_k(nl+m)n!} \tag{5}$$

Where$(\rho)_{n,k}$and $\Gamma_k(.)$Represent the k-pochhammer symbol, and the k-Gamma function respectively [14], and k>0; $z,l,m,\rho\epsilon\mathbb{C}\,,\Re(l)>0\,,\Re(m)>0,\Re(\rho)>0$

The above k-MLf is further extended and generalized by Gehlot[26]:

$$\mathrm{G}E_{k,l,m}^{\rho.\sigma}(z)=\sum_{n=0}^{\infty}\frac{(\rho)_{n\sigma,k}z^n}{\Gamma_k(nl+m)n!} \tag{6}$$

Where k $\epsilon\Re_{;}$; $z,l,m,\rho\epsilon\mathbb{C}\,,\Re(l)>0\,,\Re(m)>0,\Re(\rho)>0,\sigma\epsilon(0,1)\cup\mathbb{N}$

Many researchers have conducted extensive investigations into exploring the applications and extensions of MLF in recent years; see ([15], [16], [17], [18], [19], [20], [21]). One of them is the extended k-MLf discussed by Rahman et al. [22], in 2018 as follows:

$$E_{k,l,m}^{\rho,c}(z;P)=\sum_{n=0}^{\infty}\ \frac{B_k(\rho+nk,c-\rho,P)}{B_k(\rho,c-\rho)}\frac{(c)_{n,k}}{\Gamma_k(nl+m)}\frac{z^n}{n!}, \tag{7}$$

For $\quad\frac{(\rho)_{n,k}}{(c)_{n,k}}\quad=\frac{B_k(\rho+nk,c-\rho)}{B_k(\rho,c-\rho)}$ (8)

$B_k(x,y)$) is the k-Beta function defined in [14] and $B_k(l,m;P)$ is an extension of the k-Beta function [23], given as:

$$B_k(l,m;P)=\frac{1}{k}\int_0^1(t)^{\frac{l}{k}-1}(1-t)^{\frac{m}{k}-1}(e)^{-\frac{P^k}{kt(1-t)}}\,\mathrm{dt} \tag{9}$$

Recently, extended k-generalized Mittag –Leffler function presented by S. jain et al. [27] as follows
$E_{k,l,m}^{\rho,\sigma,c}(x,P)=\sum_{n=0}^{\infty}\ \frac{B_k(\rho+n\sigma k,c-\rho,P)}{B_k(\rho,c-\rho)}\frac{(c)_{n\sigma,k}}{\Gamma_k(nl+m)}\frac{x^n}{n!}$, where k > 0; x, l, m, ρ ∈ C; < (l) > 0, < (m) > 0,
< (ρ) > 0, σ ∈ (0, 1) ∪ N; p ≥ 0. (10)

Lavoie-Troitter integral formula

$$\int_0^1 x^{\alpha-1}\,(1-x)^{2\beta-1}(1-\tfrac{x}{3})^{2\alpha-1}(1-\tfrac{x}{4})^{\beta-1}\mathrm{d}x=\left(\frac{2}{3}\right)^{2\alpha}\left(\frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}\right),$$

(R (α) > 0 and R (β) > 0) (11)

2. Main result (Part-1):

In this section, we established some generalized integral formulae which are expressed in terms of k-Beta and k- Gamma function by inserting GKMLF(generalized K-Mittag-Leffler function) (10) with suitable arrangements into (11).

$$\int_0^1 x^{\alpha-1}\,(1-x)^{2\beta-1}(1-\tfrac{x}{3})^{2\alpha-1}(1-\tfrac{x}{4})^{\beta-1}E_{k,l,m}^{\rho,\sigma,c}\left(y(1-x)^2(1-\tfrac{x}{4})\,,P\right)\mathrm{dx}$$
$$=\left(\frac{2}{3}\right)^{2\alpha}\frac{\Gamma(\alpha)\Gamma(n+\beta)}{\Gamma(\alpha+n+\beta)}\sum_{n=0}^{\infty}\ \frac{B_k(\rho+n\sigma k,c-\rho,P)}{B_k(\rho,c-\rho)}\frac{(c)_{n\sigma,k}}{\Gamma_k(nl+m)}\frac{(y)^n}{n!} \tag{12}$$

Also eq. (12) can be expressed in other form (in integral form)

$$\int_0^1 x^{\alpha-1}\,(1-x)^{2\beta-1}(1-\tfrac{x}{3})^{2\alpha-1}(1-\tfrac{x}{4})^{\beta-1}E_{k,l,m}^{\rho,\sigma,c}\left(y(1-x)^2(1-\tfrac{x}{4})\,,P\right)\mathrm{dx}=$$

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$$\left(\frac{2}{3}\right)^{2\alpha}\sum_{n=0}^{\infty}\frac{\Gamma(\alpha)\Gamma(n+\beta)}{\Gamma(\alpha+n+\beta)}\frac{(c)_{n\sigma,k}}{\Gamma_k(nl+m)}\frac{\Gamma_k(c)}{\Gamma_k(\rho)\Gamma_k(c-\rho)}\frac{(y)^n}{n!}\frac{1}{k}\int_0^1(t)^{\frac{\rho+n\sigma k}{k}-1}(1-t)^{\frac{c-\rho}{k}-1}(e)^{-\frac{P^k}{kt(1-t)}}\,dt \quad (13)$$

Equation (12) and (13) are our main results. Now we see some special cases, for this we require some elementary relations for k-Gamma and k-Beta functions ([14],[24]):

→For,k>0,$s\in\mathbb{C}$; $\Re(s)>0$, the k-Gamma function holds the following integral representation:

$$\Gamma_k(s)=\int_0^{\infty}t^{s-1}\,e^{\frac{-t^k}{k}}\mathrm{dt} \quad (14)$$

→For, k>0, $s\in\mathbb{C}$;$\Re(s)>0$, the k-Gamma function satisfies the relation given below

$$\Gamma_k(s+k)=x\Gamma_k(s) \quad (15)$$

→For, k>0, $\Re(l)>0$, $\Re(m)>0$ the following integral representation of k-Beta function and its relation with k-Gamma function holds true:

$$B_k(l,m)=\frac{1}{k}\int_0^1(t)^{\frac{l}{k}-1}(1-t)^{\frac{m}{k}-1}\mathrm{dt}=\frac{\Gamma_k(l)\Gamma_k(m)}{\Gamma_k(l+m)} \quad (16)$$

→For, k>0, the k-Pochhammer symbol has expression in terms of
K-Gamma function

$$(x)_{n,k}=\frac{\Gamma_k(x+nk)}{\Gamma_k(x)} \quad (17)$$

→The k-Pochhammer symbol holds the addition formula as:

$$(a)_{m+n,k}=(a)_{m,k}(a+mk)_{n,k}\ ;\ k>0 \quad (18)$$

2.1Special cases:

(i) Taking σ = 1 in the eq. (3), we have extended k-MLf (7) $E_{k,l,m}^{\rho,\sigma,c}(z,P)\rightarrow E_{k,l,m}^{\rho,c}(z;P)$ and equation (12) transform into following form

$$\int_0^1 x^{\alpha-1}(1-x)^{2\beta-1}(1-\tfrac{x}{3})^{2\alpha-1}(1-\tfrac{x}{4})^{\beta-1}E_{k,l,m}^{\rho,c}\left(y(1-x)^2(1-\tfrac{x}{4}),P\right)\mathrm{dx}$$
$$=\left(\frac{2}{3}\right)^{2\alpha}\frac{\Gamma(\alpha)\Gamma(n+\beta)}{\Gamma(\alpha+n+\beta)}\sum_{n=0}^{\infty}\frac{B_k(\rho+nk,c-\rho,P)}{B_k(\rho,c-\rho)}\frac{(c)_{n,k}}{\Gamma_k(nl+m)}\frac{x^n}{n!}. \quad (19)$$

(ii) Taking σ = k = 1 in the eq. (3), we have extended MLf introduce by M. A. Ozarslan and B. Yilmaz in 2014[25].

And equation (12) transform into following form

$$\int_0^1 x^{\alpha-1}(1-x)^{2\beta-1}(1-\tfrac{x}{3})^{2\alpha-1}(1-\tfrac{x}{4})^{\beta-1}E_{l,m}^{\rho,c}\left(y(1-x)^2(1-\tfrac{x}{4}),P\right)dx$$
$$=\left(\frac{2}{3}\right)^{2\alpha}\frac{\Gamma(\alpha)\Gamma(n+\beta)}{\Gamma(\alpha+n+\beta)}\sum_{n=0}^{\infty}\frac{B\,(\rho+nk,c-\rho,P)}{B\,(\rho,c-\rho)}\frac{(c)_{n,k}}{\Gamma\,(nl+m)}\frac{x^n}{n!}. \quad (20)$$

(iii) Considering σ = 1, k = 1, p = 0 in the equation (3), we have MLf [11] defined in$E_{k,l,m}^{\rho,\sigma,c}(z,P)\rightarrow E_{l,m}^{\rho}(z)$and equation (12) transform into following form

$$\int_0^1 x^{\alpha-1}(1-x)^{2\beta-1}(1-\tfrac{x}{3})^{2\alpha-1}(1-\tfrac{x}{4})^{\beta-1}E_{l,m}^{\rho}\left(y(1-x)^2(1-\tfrac{x}{4})\right)\mathrm{dx}$$
$$=\left(\frac{2}{3}\right)^{2\alpha}\frac{\Gamma(\alpha)\Gamma(n+\beta)}{\Gamma(\alpha+n+\beta)}\sum_{n=0}^{\infty}\frac{B\,(\rho+n,c-\rho)}{B_k(\rho,c-\rho)}\frac{(c)_n}{\Gamma\,(nl+m)}\frac{x^n}{n!}. \quad (21)$$

Above three cases applied in equation (13) and transformed as follows

(iv) $setting\ \sigma=1$

$$\int_0^1 x^{\alpha-1}(1-x)^{2\beta-1}(1-\tfrac{x}{3})^{2\alpha-1}(1-\tfrac{x}{4})^{\beta-1}E_{k,l,m}^{\rho,\sigma,c}\left(y(1-x)^2(1-\tfrac{x}{4}),P\right)\mathrm{dx}=$$
$$\left(\frac{2}{3}\right)^{2\alpha}\sum_{n=0}^{\infty}\frac{\Gamma(\alpha)\Gamma(n+\beta)}{\Gamma(\alpha+n+\beta)}\frac{(c)_{n,k}}{\Gamma_k(nl+m)}\frac{\Gamma_k(c)}{\Gamma_k(\rho)\Gamma_k(c-\rho)}\frac{(y)^n}{n!}\frac{1}{k}\int_0^1(t)^{\frac{\rho+nk}{k}-1}(1-t)^{\frac{c-\rho}{k}-1}(e)^{-\frac{P^k}{kt(1-t)}}\,dt \quad (22)$$

(v) Taking σ = k = 1

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$$\int_0^1 x^{\alpha-1}(1-x)^{2\beta-1}(1-\tfrac{x}{3})^{2\alpha-1}(1-\tfrac{x}{4})^{\beta-1}E_{k,l,m}^{\rho,\sigma,c}\left(y(1-x)^2(1-\tfrac{x}{4}),P\right)\mathrm{dx} = \left(\frac{2}{3}\right)^{2\alpha}\sum_{n=0}^{\infty}\frac{\Gamma(\alpha)\Gamma(n+\beta)}{\Gamma(\alpha+n+\beta)}\frac{(c)_n}{\Gamma_k(nl+\ \ )}\frac{\Gamma(c)}{\Gamma(\rho)\Gamma(c-\rho)}\frac{(y)^n}{n!}\int_0^1(t)^{\rho+n-1}(1-t)^{c-\rho-1}(e)^{-\frac{P}{t(1-t)}}\,dt \quad (23)$$

(vi)Considering σ = 1, k = 1, p = 0

$$\int_0^1 x^{\alpha-1}(1-x)^{2\beta-1}(1-\tfrac{x}{3})^{2\alpha-1}(1-\tfrac{x}{4})^{\beta-1}E_{l,m}^{\rho}\left(y(1-x)^2(1-\tfrac{x}{4})\right)\mathrm{dx} = \left(\frac{2}{3}\right)^{2\alpha}\sum_{n=0}^{\infty}\frac{\Gamma(\alpha)\Gamma(n+\beta)}{\Gamma(\alpha+n+\beta)}\frac{(c)_n}{\Gamma(nl+m)}\frac{\Gamma(c)}{\Gamma(\rho)\Gamma(c-\rho)}\frac{(y)^n}{n!}\int_0^1(t)^{\rho+n-1}(1-t)^{c-\rho-1}\,dt \quad (24)$$

3. Main result (Part-2):

In this section, we established some more generalized integral formulae which are expressed in terms of k- Beta and k- Gamma function

$$\int_0^1 x^{\alpha-1}(1-x)^{2\beta-1}(1-\tfrac{x}{3})^{2\alpha-1}(1-\tfrac{x}{4})^{\beta-1}E_{k,l,m}^{\rho,\sigma,c}\left(yx\left(1-\tfrac{x}{3}\right)^2,P\right)\mathrm{dx} = \left(\frac{2}{3}\right)^{2(n+\alpha)}\frac{\Gamma(n+\alpha)\Gamma(\beta)}{\Gamma(\alpha+n+\beta)}\sum_{n=0}^{\infty}\frac{B_k(\rho+n\sigma k,c-\rho,P)}{B_k(\rho,c-\rho)}\frac{(c)_{n\sigma,k}}{\Gamma_k(nl+m)}\frac{(y)^n}{n!} \quad (12.1)$$

Same equation (13) transform in (13.1) as follows

$$\int_0^1 x^{\alpha-1}(1-x)^{2\beta-1}(1-\tfrac{x}{3})^{2\alpha-1}(1-\tfrac{x}{4})^{\beta-1}E_{k,l,m}^{\rho,\sigma,c}\left(yx\left(1-\tfrac{x}{3}\right)^2,P\right)\mathrm{dx} = \left(\frac{2}{3}\right)^{2(n+\alpha)}\sum_{n=0}^{\infty}\frac{\Gamma(n+\alpha)\Gamma(\beta)}{\Gamma(\alpha+n+\beta)}\frac{(c)_{n\sigma,k}}{\Gamma_k(nl+m)}\frac{\Gamma_k(c)}{\Gamma_k(\rho)\Gamma_k(c-\rho)}\frac{(y)^n}{n!}\frac{1}{k}\int_0^1(t)^{\frac{\rho+n\sigma k}{k}-1}(1-t)^{\frac{c-\rho}{k}-1}(e)^{-\frac{P^k}{kt(1-t)}}\,dt \quad (13.1)$$

3.1 Special cases:

(i) Taking σ = 1 in (12.1) and (13.1) we have extended k-MLf(7)

$$\int_0^1 x^{\alpha-1}(1-x)^{2\beta-1}(1-\tfrac{x}{3})^{2\alpha-1}(1-\tfrac{x}{4})^{\beta-1}E_{k,l,m}^{\rho,c}\left(yx\left(1-\tfrac{x}{3}\right)^2,P\right)\mathrm{dx} = \left(\frac{2}{3}\right)^{2(n+\alpha)}\frac{\Gamma(n+\alpha)\Gamma(\beta)}{\Gamma(\alpha+n+\beta)}\sum_{n=0}^{\infty}\frac{B_k(\rho+nk,c-\rho,P)}{B_k(\rho,c-\rho)}\frac{(c)_{n,k}}{\Gamma_k(nl+m)}\frac{(y)^n}{n!} \quad (25)$$

$$\int_0^1 x^{\alpha-1}(1-x)^{2\beta-1}(1-\tfrac{x}{3})^{2\alpha-1}(1-\tfrac{x}{4})^{\beta-1}E_{k,l,m}^{\rho,c}\left(yx\left(1-\tfrac{x}{3}\right)^2,P\right)\mathrm{dx} = \left(\frac{2}{3}\right)^{2(n+\alpha)}\sum_{n=0}^{\infty}\frac{\Gamma(n+\alpha)\Gamma(\beta)}{\Gamma(\alpha+n+\beta)}\frac{(c)_{n,k}}{\Gamma_k(nl+m)}\frac{\Gamma_k(c)}{\Gamma_k(\rho)\Gamma_k(c-\rho)}\frac{(y)^n}{n!}\frac{1}{k}\int_0^1(t)^{\frac{\rho+nk}{k}-1}(1-t)^{\frac{c-\rho}{k}-1}(e)^{-\frac{P^k}{kt(1-t)}}\,dt \quad (26)$$

(ii)Taking σ = k = 1 in (12.1) and (13.1)

$$\int_0^1 x^{\alpha-1}(1-x)^{2\beta-1}(1-\tfrac{x}{3})^{2\alpha-1}(1-\tfrac{x}{4})^{\beta-1}E_{l,m}^{\rho,c}\left(yx\left(1-\tfrac{x}{3}\right)^2,P\right)\mathrm{dx} = \left(\frac{2}{3}\right)^{2(n+\alpha)}\frac{\Gamma(n+\alpha)\Gamma(\beta)}{\Gamma(\alpha+n+\beta)}\sum_{n=0}^{\infty}\frac{B(\rho+nk,c-\rho,P)}{B(\rho,c-\rho)}\frac{(c)_n}{\Gamma(nl+m)}\frac{(y)^n}{n!} \quad (27)$$

$$\int_0^1 x^{\alpha-1}(1-x)^{2\beta-1}(1-\tfrac{x}{3})^{2\alpha-1}(1-\tfrac{x}{4})^{\beta-1}E_{l,m}^{\rho,c}\left(yx\left(1-\tfrac{x}{3}\right)^2,P\right)\mathrm{dx} = \left(\frac{2}{3}\right)^{2(n+\alpha)}\sum_{n=0}^{\infty}\frac{\Gamma(n+\alpha)\Gamma(\beta)}{\Gamma(\alpha+n+\beta)}\frac{(c)_n}{\Gamma(nl+m)}\frac{\Gamma(c)}{\Gamma(\rho)\Gamma(c-\rho)}\frac{(y)^n}{n!}\int_0^1(t)^{\rho+n-1}(1-t)^{c-\rho-1}(e)^{-\frac{P}{t(1-t)}}\,dt \quad (28)$$

(iii) Setting σ = 1, k = 1, p = 0

$$\int_0^1 x^{\alpha-1}(1-x)^{2\beta-1}(1-\tfrac{x}{3})^{2\alpha-1}(1-\tfrac{x}{4})^{\beta-1}E_{l,m}^{\rho,c}\left(yx\left(1-\tfrac{x}{3}\right)^2\right)\mathrm{dx} = \left(\frac{2}{3}\right)^{2(n+\alpha)}\frac{\Gamma(n+\alpha)\Gamma(\beta)}{\Gamma(\alpha+n+\beta)}\sum_{n=0}^{\infty}\frac{B(\rho+n,c-\rho)}{B(\rho,c-\rho)}\frac{(c)_n}{\Gamma(nl+m)}\frac{(y)^n}{n!} \quad (29)$$

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$$\int_0^1 x^{\alpha-1}\,(1-x)^{2\beta-1}(1-\tfrac{x}{3})^{2\alpha-1}(1-\tfrac{x}{4})^{\beta-1}E_{l,m}^{\rho,c}\left(\mathrm{yx}\left(1-\tfrac{\mathrm{x}}{3}\right)^{2}\right)\mathrm{dx} =$$

$$\left(\frac{2}{3}\right)^{2(\mathrm{n}+\alpha)}\sum_{n=0}^{\infty}\frac{\Gamma(n+\alpha)\Gamma(\beta)}{\Gamma(\alpha+n+\beta)}\frac{(c)_n}{\Gamma\,(nl+m)}\frac{\Gamma\,(c)}{\Gamma\,(\rho)\Gamma\,(c-\rho)}\frac{(y)^n}{n!}\int_0^1(t)^{\rho+n-1}(1-t)^{c-\rho-1}dt \quad (30)$$

4. Conclusion:

In this paper, new generalized integral formulae involving the generalized special functions are obtained,some interesting special cases of the main results are also considered. In future more integral transform formulae will be used on above functions to find out more interesting results.

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