CAHIERS MAGELLANES-NS
Volume 06 Issue 2
2024

ISSN:1624-1940
DOI 10.6084/m9.figshare.2632574
http://magellanes.com/

A STUDY ON ZAGREB ENERGY OF AN UNDIRECTED GRAPHS G_n And ${G_{m,n}}^M$

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Abstract: Let G be a simple graph of order n with vertices $v_1, v_2, .. v_n$ and the corresponding degrees of the vertices are $d_1, d_2, .. d_n$.The First Zagreb energy of the graph G is defined as the sum of the absolute Zagreb eigen values of First Zagreb matrix $A_z(G)$ of a graph G.In this paper First Zagreb energy of undirected graphs ${G_{m,n}}^M$ and G_n graphs are computed.

Keywords: Zagreb energy of a graph , ${G_{m,n}}^M -$ graph, G_n- graph.

1. Introduction

Zagreb indices was introduced by Gutman[1-2] in 1972 .The Zagreb matrix of a graph G is defined in [3].Some classes of Zagreb hyperenergetic, Zagreb equienergetic graphs are studied in[4]. Upper and lower bounds for zagreb energy are studied in[5].The zagreb energy for some classes of graphs studied in [6].The relation between second zagreb matrix and the adjacency matrix derived in [7].The Zagreb indices of selected chemical compounds of natural products are studied in Siva Parvathi et al [8].The Zagreb indices of Arithmetic graphs are studied by Anusha et al.[9]

The undirected graphs G_n and ${G_{m,n}}^M$ are defined and proved some properties by Ivy Chakrabarthy et al [10],[11].More on Zagreb indices are studied in [12-16].Some energies of ${G_{m,n}}^M$ graph and G_n graph calculated in [17] . Motivated by these in this paper we calculate the first Zagreb energy of undirected ${G_{m,n}}^M$ graph and undirected G_n graphs. .

Let G be a simple graph with vertex set $V(G) = \{v_1, v_2, ... v_n\}$ and let d_i be the degree of the vertex v_i. $E(G)$ be the edge set of G.The First Zagreb matrix $A_z(G)$ of a graph G is a square matrix of order n and is defined as

$A_Z(G) = [m_{ij}]$ of order n where $m_{ij} = \begin{Bmatrix} d_i + d_j, if\, v_i \, and \, v_j \, are \, adjacent \\ 0, \quad otherwise \end{Bmatrix}$

Let $z_1, z_2, z_3, ... z_n$ are the Zagreb eigen values of $A_Z(G)$ of a graph G with $z_1 \geq z_2 \geq \cdots \geq z_n$ having multiplicities $m_1, m_2, m_3, ... m_n$ then Zagreb spectrum of graph G is denoted as

Spec $(A_Z(G)) = \begin{pmatrix} z_1 & \dots & z_n \\ m_1 & \dots & m_n \end{pmatrix}$ where $m_1 + m_2 + \cdots m_n = n$.

The first Zagreb energy of the graph G is defined as the sum of the absolute Zagreb eigen values of $A_Z(G)$ of a graph G and is denoted by $E_Z(G)$ where $E_Z(G) = \sum_{i=1}^{n} |z_i|$.

2. First Zagreb energy of the undirected graph G_n

Ivy chakrabarthy[10] defined an Undirected graph G_n on a natural numbers finite subset and discussed the properties of graph G_n. Let an undirected simple graph $G_n = (V, E)$ whose vertex set V is a subset of Natural numbers defined as $V = \{x \in N/(x, n) \neq 1, x < n\}$, where $n \in N$ and n is not a

ISSN:1624-1940
DOI 10.6084/m9.figshare.2632574
http://magellanes.com/

prime number and $x, y \in V$ are two adjacent vertices if and only if gcd $(x, y) > 1$.The following are the some of the properties of G_n.

1. The G_n graph is complete if and only if $n = p^m$ where p is prime.
2. The G_n graph is disconnected if and only if $n = 2p$ where p is an odd prime.

First Zagreb energy of the G_n graph is presented in this section.

Theorem 2.1: The First Zagreb energy of the $G_n$graph, where $n = 2^\alpha, \alpha > 1$ is $(2^\alpha - 4)^2$.

Proof: Let $n = 2^\alpha, \alpha > 1$ then the vertex set of $G_n$graph is V={2, 2.2, 3.2, ….. $(2^{\alpha-1} - 1).2\}$. By the G_n graph definition, if $x, y \epsilon V$ are adjacent then $\gcd(x, y) > 1$.

The First Zagreb matrix of G_{2^α} graph is

$$A_z(G_{2^\alpha}) = (2^\alpha - 4)\begin{pmatrix} 0 & 1 & 1 & \dots & 1 \\ 1 & 0 & 1 & \dots & 1 \\ 1 & 1 & 0 & \dots & 1 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & 1 & \dots & 0 \end{pmatrix}_{2^{\alpha-1}-1\times 2^{\alpha-1}-1}$$

The Characteristic Equation of $A_z(G_{2^\alpha})$ of graph G_{2^α} is

$$(\omega + (2^\alpha - 4))^{\frac{(2^\alpha-4)}{2}}(\omega - \frac{(2^\alpha-4)^2}{2}) = 0.$$

Then the eigen values of $A_z(G_{2^\alpha})$ are $(2^\alpha - 4)$ and $\frac{(2^\alpha-4)^2}{2}$ with corresponding multiplicities $\frac{(2^\alpha-4)}{2}$ and 1. Hence the spectrum of the G_{2^α} graph is $\begin{pmatrix} -(2^\alpha - 4) & \frac{(2^\alpha-4)^2}{2} \\ \frac{(2^\alpha-4)}{2} & 1 \end{pmatrix}$.

Then the first Zagreb energy of the graph $G_{2^\alpha}$is $E_Z(G_{2^\alpha}) = |-(2^\alpha - 4)|\frac{(2^\alpha-4)}{2} + \left|\frac{(2^\alpha-4)^2}{2}\right|(1) = (2^\alpha - 4)^2$.

3. First Zagreb energy of an undirected graph $G_{m,n}{}^M$

Ivy Chakrabarthy [11] introduced the Undirected simple graph $G_{m,n}{}^M$ and proved properties of $G_{m,n}{}^M$graph.Let $G_{m,n}{}^M = (V, E)$ on natural numbers finite subset and $m, n \in N$,where the vertex set $V = \{1,2, \dots . n\}$ and two distinct vertices $u, v \in V$ are adjacent if and only if $u \neq v$and $u.v$ is not divisible by m. Some of the properties are

1. Let $m = 1$ then the graph $G_{m,n}{}^M$ is a null graph with n vertices.
2. For $1 < m \leq n$, the graph $G_{m,n}{}^M$ is disconnected.
3. The graph $G_{m,n}{}^M$is connected for $m > n$.
4. The graph $G_{m,n}{}^M$ has the maximum degree $n - 1$.

The First Zagreb energy of the graph $G_{m,n}{}^M$ is presented in this section.

Theorem 3.1: The First Zagreb energy of the $G_{m,n}{}^M$ graph when $n = 2p$,$m > n$, m, p are primes is $4(2p - 1)^2$.

Proof: Consider an undirected $G_{m,n}{}^M$ graph when $m > n, n = 2p$ and m, p are primes with the vertex

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2024
ISSN:1624-1940
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http://magellanes.com/

set $V = \{1,2,3 \ldots n\}$.
The First Zagreb matrix of ${G_{m,2p}}^M$ graph is

$$A_z\left({G_{m,2p}}^M\right) = 2(2p-1)\begin{pmatrix} 0 & 1 & 1 & \ldots & 1 \\ 1 & 0 & 1 & \ldots & 1 \\ 1 & 1 & 0 & \ldots & 1 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & 1 & \ldots & 0 \end{pmatrix}_{2p\times 2p}$$

The characteristic equation of $A_z\left({G_{m,2p}}^M\right)$ of ${G_{m,2p}}^M$ graph is $\left(\omega + 2(2p-1)\right)^{2p-1}(\omega - 2(2p-1)^2) = 0$.
Then the eigen values of $A_z\left({G_{m,2p}}^M\right)$ of ${G_{m,2p}}^M$ graph are $-2(2p-1), 2(2p-1)^2$ with $2p-1$ and 1 are corresponding multiplicities. Hence the spectrum of the ${G_{m,2p}}^M$ graph is $\begin{pmatrix} -2(2p-1) & 2(2p-1)^2 \\ 2p-1 & 1 \end{pmatrix}$. The First Zagreb energy of the ${G_{m,2p}}^M$ graph is $E_Z\left({G_{m,2p}}^M\right) = |-2(2p-1)|(2p-1) + |2(2p-1)^2|(1) = 4(2p-1)^2$.

Theorem 3.2: The First Zagreb energy of the ${G_{m,n}}^M$ graph when $m > n$ and m, n are primes is $4(n-1)^2$. .

Proof: Consider an undirected ${G_{m,n}}^M$ graph, $m > n$ and m, n are primes with the vertex set $V = \{1,2,3 \ldots n\}$. The First Zagreb matrix of ${G_{m,n}}^M$ graph is

$$A_z\left({G_{m,n}}^M\right) = 2(n-1)\begin{pmatrix} 0 & 1 & 1 & \ldots & 1 \\ 1 & 0 & 1 & \ldots & 1 \\ 1 & 1 & 0 & \ldots & 1 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & 1 & \ldots & 0 \end{pmatrix}_{n\times n}$$

The characteristic equation of $A_z\left({G_{m,n}}^M\right)$ of graph ${G_{m,n}}^M$ is $\left(\omega + 2(n-1)\right)^{n-1}(\omega - 2(n-1)^2) = 0$.
Then the eigen values of $A_z\left({G_{m,n}}^M\right)$ of ${G_{m,n}}^M$ graph are $-2(n-1), 2(n-1)^2$ with $n-1$ and 1 are corresponding multiplicities. Hence the spectrum of the ${G_{m,n}}^M$ graph is $\begin{pmatrix} -2(n-1) & 2(n-1)^2 \\ n-1 & 1 \end{pmatrix}$. The First Zagreb energy of the ${G_{m,n}}^M$ graph is $E_Z\left({G_{m,n}}^M\right) = |-2(n-1)|(n-1) + |2(n-1)^2|(1) = 4(n-1)^2$.

Theorem 3.3: The First Zagreb energy of the ${G_{m,n}}^M$ graph when $m > n$, $n = 2^\alpha$ and m is prime is $4(2^\alpha - 1)^2$. .

Proof: Consider an undirected ${G_{m,n}}^M$ graph, $m > n$, $n = 2^\alpha$ and m is prime with the vertex set $V = \{1,2,3 \ldots n\}$. The First Zagreb matrix of ${G_{m,2^\alpha}}^M$ graph is

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$$A_z\left({G_{m,2^\alpha}}^M\right) = 2(2^\alpha - 1)\begin{pmatrix} 0 & 1 & 1 & \dots & 1 \\ 1 & 0 & 1 & \dots & 1 \\ 1 & 1 & 0 & \dots & 1 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & 1 & \dots & 0 \end{pmatrix}_{2^\alpha \times 2^\alpha}$$

The characteristic equation of$A_z\left({G_{m,2^\alpha}}^M\right)$ of graph ${G_{m,2^\alpha}}^M$ is

$\left(\omega + 2(2^\alpha - 1)\right)^{2^\alpha - 1}(\omega - 2(2^\alpha - 1)^2) = 0.$ Then the eigen values of $A_z\left({G_{m,2^\alpha}}^M\right)$of ${G_{m,2^\alpha}}^M$ graph are $-2(2^\alpha - 1)$, $2(2^\alpha - 1)^2$ with $2^\alpha - 1$ and 1 are corresponding multiplicities.

The spectrum of the ${G_{m,2^\alpha}}^M$ graph is $\begin{pmatrix} -2(2^\alpha - 1) & 2(2^\alpha - 1)^2 \\ 2^\alpha - 1 & 1 \end{pmatrix}$. The First Zagreb energy of the ${G_{m,2^\alpha}}^M$ graph is $E_Z\left({G_{m,2^\alpha}}^M \right) = |-2(2^\alpha - 1)|(2^\alpha - 1) + |2(2^\alpha - 1)^2|(1) = 4(2^\alpha - 1)^2$.

4. Conclusion

The graph energy plays a vital role in computations of molecular graphs in chemistry .The First Zagreb energy of the undirected graphs $\boldsymbol{G_n}$ and ${G_{m,n}}^M$ for some cases are computed in this paper.

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