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MODELLING LATENT INFECTED DYNAMICS IN TUBERCULOSIS SPREAD IN INDIAN CONTEXT

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Introduction

Tuberculosis (TB) is an infectious disease caused by the bacterium *Mycobacterium tuberculosis*. While it predominantly affects the lungs, it can also affect other organs, including the kidneys, spine, and brain. The bacteria spreads through airborne droplets expelled when an infected person coughs, sneezes, or talks. Many individuals who contract the bacteria might also develop a latent TB infection, during which the bacteria remains dormant and non-contagious. However, some may progress to active TB, characterized by symptoms such as a persistent cough, chest pain, coughing up blood, fatigue, weight loss, fever, and night sweats. Early diagnosis and treatment is essential for curing the disease and preventing transmission.

Despite medical advancements, TB remains a significant global health concern, particularly in low and middle income countries. In 2020, TB affected approximately 10 million people worldwide, resulting in 1.5 million deaths, positioning it as one of the leading causes of mortality [1][2]. The emergence of drug-resistant strains, such as multi-drug-resistant TB (MDR-TB) and extensively drug-resistant TB (XDR-TB), poses additional challenges for TB control. Key preventive measures include early detection and treatment, vaccination with Bacillus Calmette Guerin (BCG), and addressing social determinants of health. The World Health Organization (WHO) has set ambitious targets to reduce TB incidence and mortality, aiming to end the global TB epidemic by 2030, as outlined in the Sustainable Development Goals (SDGs) [3][4][5].

Compartmental models are a fundamental approach in mathematical modelling, particularly useful in epidemiology for studying the spread of infectious diseases. These models divide the population into distinct compartments based on disease status, such as susceptible, exposed, infected, and recovered individuals. By using differential equations to describe the rates of movement between compartments, we can study how diseases propagate through populations over time. Mathematical models are invaluable in understanding TB dynamics and shaping public health strategies. This paper introduces a mathematical model for TB that incorporates the latent TB component, offering a more comprehensive view of TB transmission dynamics with respect to India.

Tuberculosis Statistics in India

India bears the highest burden of TB globally, facing unique and complex public health challenges. The high incidence and mortality rates are compounded by the prevalence of multi-drug-resistant TB strains and significant barriers to healthcare access. Understanding detailed TB statistics in India, compared to other high-burden countries, is crucial for addressing and managing this critical health issue effectively. Analysing these statistics not only highlights the severity of the problem but also provides a basis for assessing the effectiveness of current public health interventions and strategies. The following

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comparative perspective shows India's standing in the global effort against TB, relative to other nations facing similar challenges.

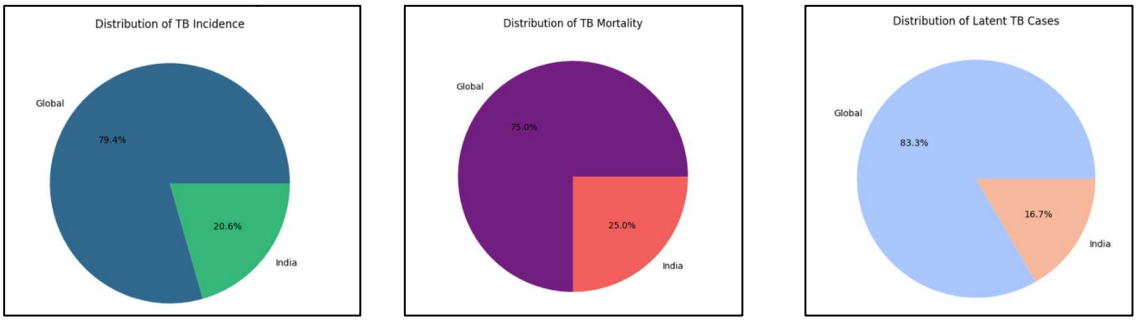

Figure1: Distribution of TB incidence, mortality and latent cases

This analysis not only highlights the scale of the problem but also frames the context for evaluating the effectiveness of ongoing public health interventions and strategies. Following is the comparative TB incidence over time. We can observe that the cases are decreasing but not significantly.

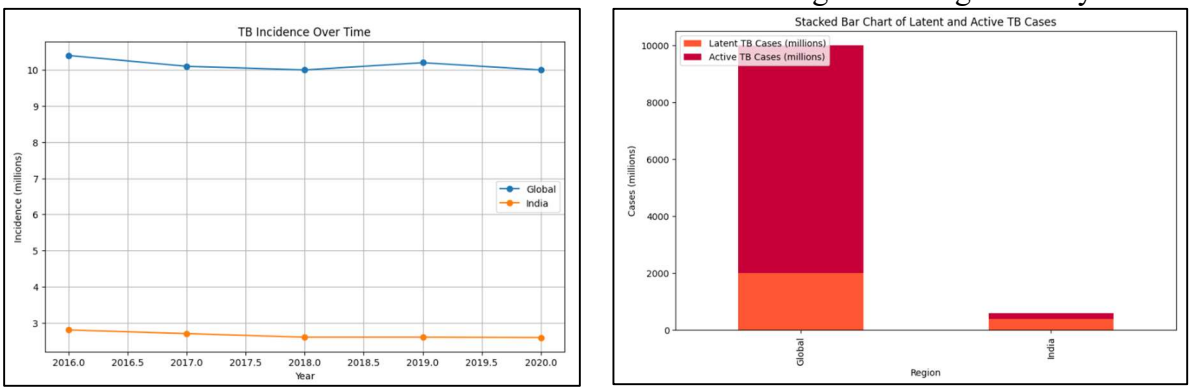

Figure 2: TB incidence over time

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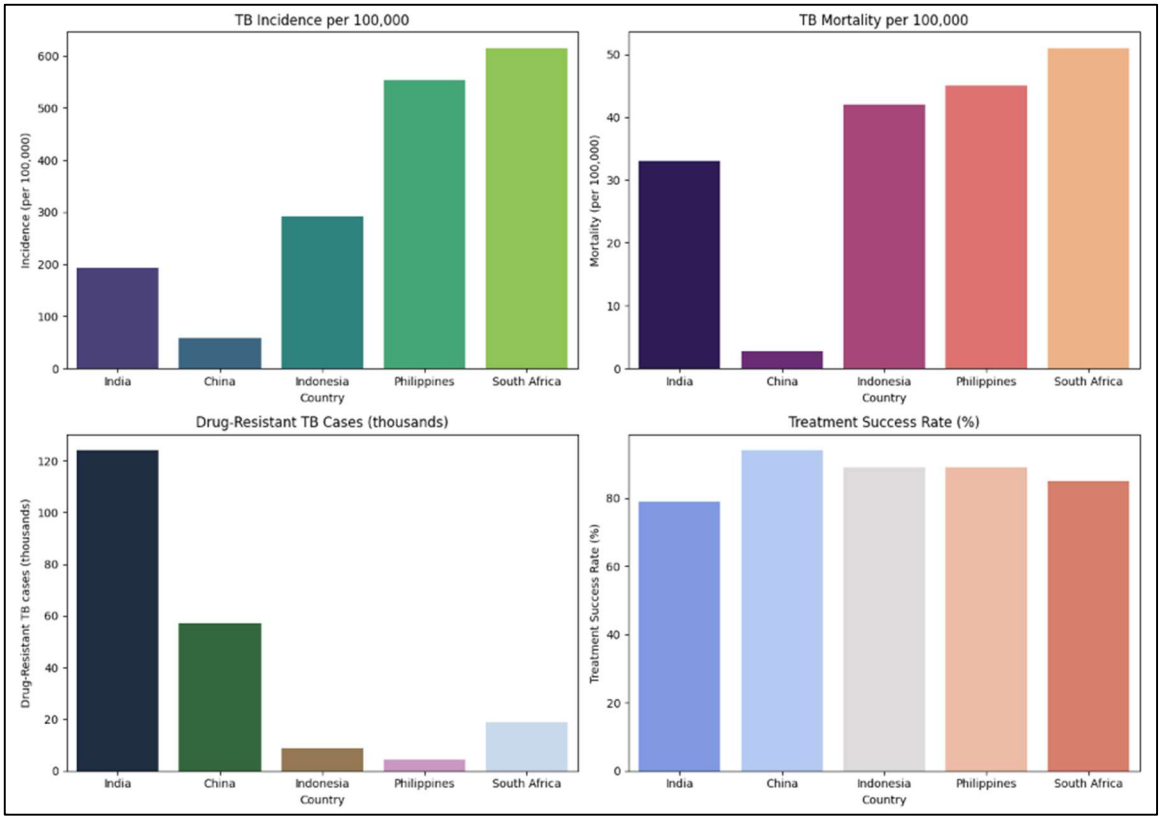

Figure 3: TB overview among different countries

Description of the Tuberculosis (TB) Dynamic Model

The dynamic model for tuberculosis (TB) presented here is a compartmental model that divides the population into five distinct compartments based on disease status. This model helps in understanding the transmission dynamics of TB and evaluating the impact of different factors on its spread. The compartments are:

- Susceptible (S): Individuals who are not infected with TB but are at risk of infection.
- Exposed (E): Individuals who have been infected with TB bacteria but are not yet infectious. This stage represents recent infections where individuals are in the incubation period.
- Latent Infected (L): Individuals who are infected with TB bacteria but do not show symptoms and are not infectious. These individuals have a latent TB infection and may later progress to active TB.
- Infectious (I): Individuals who have developed active TB disease and can transmit the infection to others. These individuals are symptomatic and contagious.
- Recovered (R): Individuals who have recovered from TB infection and are no longer infectious. They may have developed immunity against reinfection.

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Susceptible (S)

$$\frac{dS}{dt} = \mu N - \beta \frac{SI}{N} - \mu S \tag{1}$$

Exposed (E)

$$\frac{dE}{dt} = \beta \frac{SI}{N} - (\sigma + \epsilon + \mu)E \tag{2}$$

Latent Infected (L)

$$\frac{dL}{dt} = \sigma E - (\delta + \mu)L \tag{3}$$

Infectious (I)

$$\frac{dI}{dt} = \delta L + \epsilon E - (\gamma + \mu)I \tag{4}$$

Recovered (R)

$$\frac{dR}{dt} = \gamma I - \mu R \tag{5}$$

Where:

- β is the transmission rate, representing the rate at which susceptible individuals become infected upon contact with infectious individuals.
- σ is the rate at which exposed individuals progress to the latent infected state.
- δ is the rate at which latent infected individuals progress to the infectious state.
- ϵ is the rate at which exposed individuals progress directly to the infectious state.
- γ is the recovery rate, representing the rate at which infectious individuals recover and move to the recovered state.
- μ is the natural death rate, representing the rate at which individuals die from natural causes.
- N is the total population, given by N=S+E+L+I+R.

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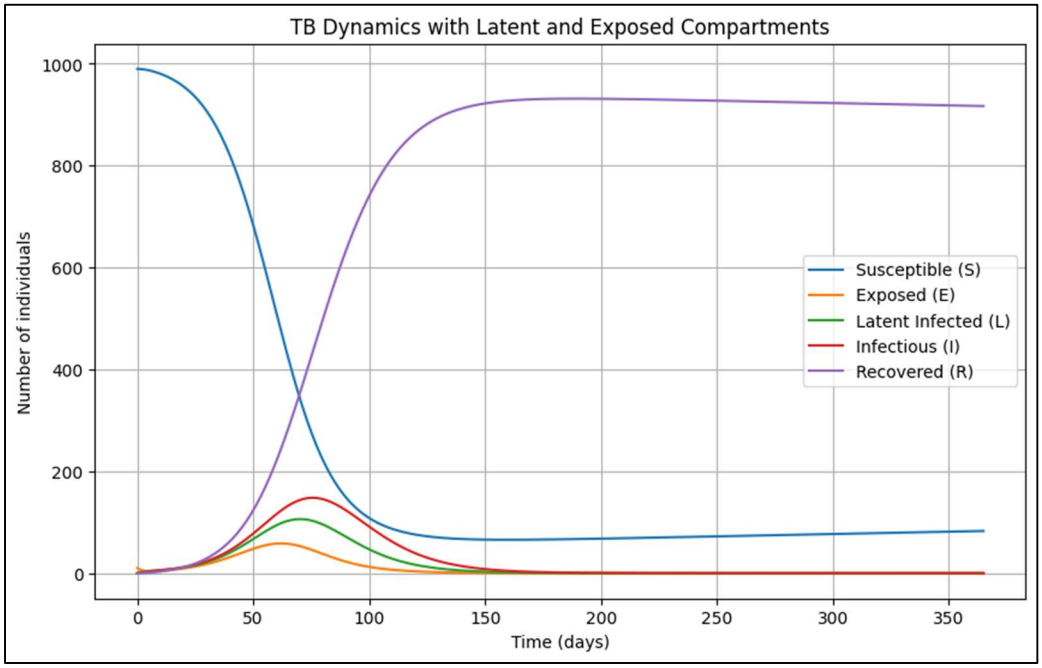

Equilibrium Points and Stability Analysis

Equilibrium points were calculated to understand the steady-state behaviour of the system. The results are as follows:

- Equilibrium Point 1: (E,I,L,R,S) = (0,0,0,0,1000)
  - Eigenvalues: {0, -0.3001, -0.2733, 0.0731, -0.0001}
  - Stability: The equilibrium point is unstable.
- Equilibrium Point 2: (E,I,L,R,S) = (0.0006*S, 0.0019*S, 0.0013*S, 1.9900*S, S: 1000 - 1.994*S}
  - Eigenvalues: {-0.3001, -0.20025, -0.0001 - 0.0031*I, -0.0001 + 0.0031*I, -1.5055×10^{-21}}
  - Stability: The equilibrium point is stable.

Sensitivity Analysis, Reproduction Number, and Simulation Results

The sensitivity analysis, performed on key parameters such as the transmission rate (β), recovery rate (γ), and natural death rate (μ), highlighted the significant impact of these parameters on the basic reproduction number (R_0) and the stability of the equilibrium points. The analysis revealed that R_0 is highly sensitive to changes in β, emphasizing the importance of infection control measures. Specifically, an increase in β results in a higher R_0, while an increase in γ decreases R_0, underscoring the critical role of effective treatment strategies.

Simulation results showed the dynamic progression of TB within the population. By evaluating the eigenvalues of the Jacobian matrix at each equilibrium point, the stability of these points was determined. The presence of negative real parts in the eigenvalues indicated stable equilibrium points, while positive real parts suggested instability. These results provide crucial insights into TB persistence or elimination under various scenarios, guiding effective intervention strategies.

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Conclusion

The mathematical modelling of tuberculosis (TB) dynamics, incorporating the latent infected compartment, provides valuable insights into the transmission and progression of TB within a population, particularly in the Indian context. This model highlights the critical importance of addressing latent TB infections, which form a significant reservoir for future active TB cases. The inclusion of the latent compartment allows for a more comprehensive understanding of TB dynamics, emphasizing the need for targeted interventions that can prevent the progression from latent to active TB. Strategies such as early detection, preventive treatment, and vaccination can significantly reduce the incidence of active TB, thereby mitigating the overall burden of the disease.

India, with its high burden of TB and significant challenges posed by multi-drug-resistant TB (MDR-TB) and extensively drug-resistant TB (XDR-TB), stands to benefit greatly from the insights provided by this model. Implementing robust public health measures, improving healthcare access, and addressing social determinants of health are critical steps toward achieving the World Health Organization's goal of ending the global TB epidemic by 2030.

In conclusion, the compartmental TB model developed in this study serves as a powerful tool for understanding and combating TB in India. By integrating mathematical modeling with public health strategies, we can enhance our efforts to control TB, reduce its incidence and mortality, and ultimately move closer to a TB-free world. Continued research and collaboration among healthcare professionals, researchers, and policymakers are essential to achieving these goals and ensuring the health and well-being of populations worldwide.

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